Rank and Sparsity in Language Processing

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Abstract

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Language modeling is one of many problems in language processing that have to grapple with naturally high ambient dimensions. Even in large datasets, the number of unseen sequences is overwhelmingly larger than the number of observed ones, posing clear challenges for estimation. Although existing methods for building smooth language models tend to work well in general, they make assumptions that are not well suited to training with limited data.

This thesis introduces a new approach to language modeling that makes different assumptions about how best to smooth the distributions, aimed at better handling the limited data scenario. Among these, it assumes that some words and word sequences behave similarly to others and that similarities can be learned by parameterizing a model with matrices or tensors and controlling the matrix or tensor rank. This thesis also demonstrates that sparsity acts as a complement to the low rank parameters: a low rank component learns the regularities that exist in language, while a sparse one captures the exceptional sequence phenomena. The sparse component not only improves the quality of the model, but the exceptions identified prove to be meaningful for other language processing tasks, making the models useful not only for computing probabilities but as tools for the analysis of language.

Three new language models are introduced in this thesis. The first uses a factored low rank tensor to encode joint probabilities. It can be interpreted as a "mixture of unigrams"

model and is evaluated on an English genre-adaptation task. The second is an exponential model parameterized by two matrices: one sparse and one low rank. This “Sparse Plus Low Rank Language Model” (SLR-LM) is evaluated with data from six languages, finding consistent gains over the standard baseline. Its ability to exploit features of words is used to incorporate morphological information in a Turkish language modeling experiment, with some improvements over a word-only model. Lastly, its use to discover words in an unsupervised fashion from sub-word segmented data is presented, showing good performance in finding dictionary words. The third model extends the SLR-LM in order to capture diverse and overlapping influences on text (e.g. topic, genre, speaker) using additive sparse matrices. The “Multi-Factor SLR-LM” is evaluated on three corpora with different factoring structures, showing improvements in perplexity and the ability to find high quality factor-dependent keywords. Finally, models and training algorithms are presented that extend the low rank ideas of the thesis to sequence tagging and acoustic modeling.

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DEDICATION

to my wife, Britt, my daughter, Violet, and my parents, Paul and Kathy

Chapter 1
INTRODUCTION

1.1 Problem Context

A recurring theme in speech and language processing is the need to make simplifying modeling assumptions to overcome the natural, vast ambient dimension in which language lives. For example, there are a staggering number of possible word sequences (exponential in the sequence length), so a naive maximum likelihood estimate of sequence probabilities is doomed to be poorly estimated and simplifying assumptions (e.g. that word sequences are Markov) must be made. When modeling speech acoustics, it is observed that the phonetic context (e.g. triphone or quinphone) affects the acoustic properties of a phone (speech sound), suggesting that context should be taken into account when modeling acoustics. However, the large number of tri- or quinphones again necessitates a simplification (e.g. clustering context-dependent phones) in order for the model to be adequately trained (achieve good performance given the training data). These are just two examples of the competing goals of accuracy and trainability, where the former criterion argues in favor of rich models and the latter demands simple models. In this thesis we will propose a new approach to balancing this trade-off, focusing on the problem of sequence modeling, that draws some insights from multi-task learning.

1.1.1 Sequence Modeling Overview

Language is fundamentally a sequential process. Spoken words are sequences of articulatory gestures. Written words are sequences of characters. Phrases and sentences are sequences of words. Paragraphs and their spoken equivalents are sequences of sentences and phrases, which are in turn arranged into larger discourse structures. Thus the statistical modeling of sequences is core to the automatic processing of human language. Formally, if $x_1, x_2, \ldots, x_T$ denote a sequence of some unit of language (e.g. words), the ability to assign probabilities

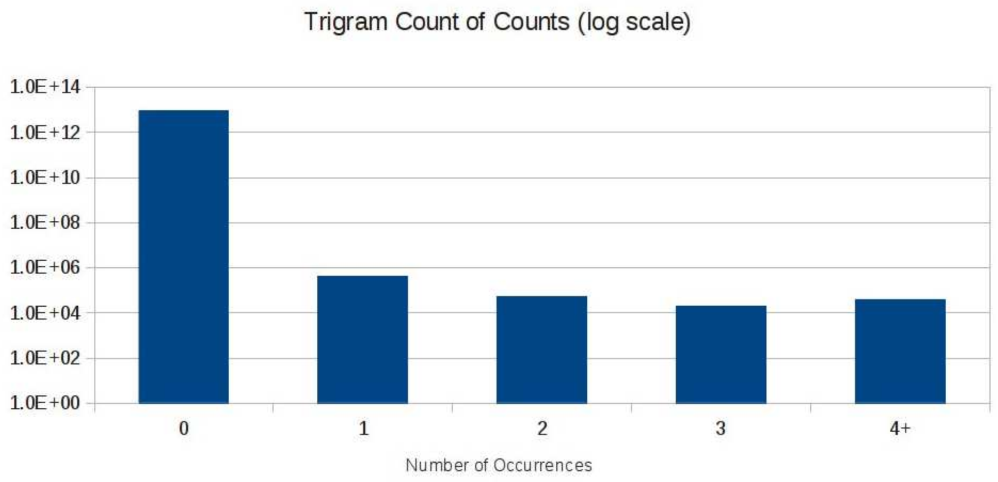


Figure 1.1: 99.99999% of possible Tagalog trigrams have zero count in a 539k word training set with a vocabulary of 20.7k words. The y-axis plots how many trigrams occur n times in the training set, where n is specified on the x-axis. The data comes from IARPA Babel data collection release babel106b-v0.2g-sub-train.

$P(x_1, x_2, \ldots, x_T)$ is critical. We focus on the case where they are categorical, drawn from some alphabet of size V. Such models are typically referred to as *language models*. If the maximum sequence length is fixed to T, the number of entries in the joint probability table is $O(V^T)$. Due to the exponential model complexity, even moderate values of V and T pose substantial problems for the proper estimation of these probabilities. In practice, then, one must make simplifying assumptions about the form of the distribution in order to control model complexity. As an example, the n-th order Markov assumption is widely used in language modeling, where the distributions are conditioned on only the n most recent history symbols. Even with this n-th order assumption, observations are overwhelming sparse. To illustrate, Fig. 1.1 plots the number of trigrams (sequences of three words) that occur zero, one, two, three and four or more times in a 539k word training dataset. Despite the Markov assumption, 99.99999% of trigram never occur, and 99.9999999% do not occur more than three times.

1.1.2 Language Processing as Multi-Task Learning

It is the connection between the sequence modeling problem and the field of multi-task learning that inspired the approach taken in this thesis, and it is illustrative to discuss this connection. The intuitive idea underlying multi-task learning is that it is better to solve related problems jointly than to solve them independently. If you can exploit commonalities between problems, you bring more information to bear in each individual learning task. One common mechanism for sharing information between related tasks is to learn a shared representation - either of the input features or of the class label. Although rarely viewed this way, language modeling can be naturally phrased as a multi-task learning problem.

In a language model, one must estimate (either explicitly or implicitly) a set of conditional probability distributions, $p(x|h)$, one distribution for each unique history (where "history" denotes the sequence of words that precede x). If there are H unique histories, this can be phrased as solving H different (sub-)problems. But clearly these problems are related. For example, the distributions $p(x|\text{"she wore fuchsia socks and"})$ and $p(x|\text{"she wore magenta socks and"})$ should be very similar, while the distributions $p(x|\text{"she said you would"})$ and $p(x|\text{"he insisted you could"})$ should be somewhat similar, and so forth. Though slightly less intuitive, one can instead decompose the language modeling problem as solving V related problems: finding the values $p(x|\cdot)$ for each unique word x in a vocabulary of size V. Again, the problems are clearly related since, for example, the values $p(\text{"monday"}|\cdot)$ and $p(\text{"tuesday"}|\cdot)$ should be fairly similar. As we will see, it is possible to elegantly exploit both sources of commonality between related problems by learning shared, low-dimensional representations of words and of histories. Although the standard language model strategies of backoff or interpolation (i.e. incorporating lower order Markov models) are in fact a simple, prescribed strategy for "solving related problems jointly," learning a continuous representation is much more flexible, allowing the model to discover underlying semantic and syntactic commonalities without being forced to use the back-off structure as a proxy.

1.2 Research Goals and Approach Overview

The primary goal of this work is to design novel probabilistic models of language that:

- more effectively balance the trade-off between rich (accurate) and simple (trainable) models by exploiting the inherent commonalities that exist between modeling sub-problems, and

- identify underlying structure in language, thus serving as a tool for its analysis.

We approach the task of language modeling by encoding relationships between symbols in a sequence with a matrix or tensor. This gives us a principled way to control the complexity of their relationships, so that model expressiveness may be more precisely balanced with the amount of training data available. Sparsity, on the other hand, is particularly well suited to modeling *exceptional* phenomena, such as multi-word sequences which function as a single word (e.g. "san francisco"). We will consider both a non-convex method, in which a basis of simple models is learned, and several convex methods, which learn continuous representations of words and histories, and whose training techniques possess desirable theoretical and algorithmic properties. Our approach has two key aspects:

1. We build probabilistic models that make use of multilinear functions of the input on which we are conditioning (e.g. word history) and the label we are predicting (e.g. the next word). In practice, this means using (possibly sums of) matrix or tensor weights and permits the model to capture richer (and more accurate) relationships between input and output than are supported by existing standard methods.

2. We constrain or regularize the parameter matrices or tensors to be low rank to identify and exploit commonalities between sub-problems: both can be viewed as inducing shared representations. Optionally, we regularize additional weight matrices or tensors to be sparse. The result is a simple, trainable model.

As will be seen in this thesis, this strategy leads to highly interpretable models, which can be leveraged to benefit other language processing tasks. In this thesis, we focus on the sequence modeling problem, which is fundamentally important to language processing, but include discussion of extensions to the sequence tagging acoustic modeling problems.

1.3 Major Contributions

The major contributions of this thesis, described in detail later, can be summarized as:

1. The introduction of a new low rank tensor language model, which can be interpreted as a mixture of unigram models and permits fine-grained control over model complexity.

2. The introduction of new exponential language models, parameterized by a low rank matrix and one or more sparse matrices, which

    - Automatically learn low dimensional continuous representations of words and word sequences.
    - Automatically identify meaningful, “exceptional” sequences.
    - Have a convex training objective and efficient training algorithms.
    - Can exploit features of words and word sequences beyond n-gram indicators; for example, morphological features.

3. A study of exceptional word sequences identified by the novel exponential language models, including applications to

    - Factoring out distinct, potentially overlapping factors influencing n-gram probabilities (e.g. keywords associated with topic, genre, speaker or role).
    - Automatically learning words and multiwords from syllable-segmented data.

1.4 Thesis Organization

The remainder of this thesis is organized as follows.

- In Chapter 2, we introduce the mathematical background, including a formal definition of the notions of matrix and tensor rank that will be used, as well as the basic strategies for rank minimization. We also discuss relevant prior language modeling work, including exponential and neural network models, each of which shares some properties with our models.

- In Chapter 3, we show that popular, standard language model smoothing methods have the effect of approximately reducing the rank of conditional probability matrices. Using this insight, we introduce a non-convex, factored low-rank tensor approach to language modeling, which can be interpreted as a mixture of unigram models.

- In Chapter 4, we introduce another language model which deviates from Chapter 3 in two important ways: first, it uses rank in the parameter space (of an exponential language model) rather than the probability distributions, and second, it introduces a sparse weight matrix to capture *exceptions*. We show that it is superior to standard baseline methods on experiments in several language, that it can benefit by incorporating additional (e.g. morphological) features and that the exceptions themselves can be useful for learning words and multiwords in subword-segmented languages.

- In Chapter 5, we explore how an extension of the model of Chapter 4 can be an effective tool for accounting for distinct and overlapping influences on the words in a text. We see that sparse exceptions can be an effective mechanism for identifying multi-token sequences (words or multiwords) that are particularly important to topic, role and the speaker.

- In Chapter 6, we show that the ideas behind the model of Chapter 4 can be extended in two key ways. First, the low rank matrix parameterization is extended to a low n-rank tensor parameterization, and second, we employ the ideas in the context of two different modeling problems, sequence tagging and acoustic modeling. The models and training algorithms are presented, but since they are not yet implemented, we do not include any experimental results.

- Finally, in Chapter 7, we discuss potential future extensions of this work and conclude.

Chapter 2
BACKGROUND

This chapter introduces the mathematical notation and concepts used in later chapters and then discusses the relevant prior literature into which the contributions of this thesis will be placed.

2.1 Mathematical Background

The rank of a matrix or tensor is fundamentally a measurement of the complexity of its structure: low rank matrices/tensors can be decomposed into the sum of a small number of underlying factors, while those with high rank require many. This section reviews the concepts of matrix and tensor rank and discusses the class of optimization problems that aim to minimize them.

2.1.1 Matrix Rank

There are a number of equivalent definitions of the rank of a matrix $M \in \mathbb{R}^{m \times n}$. The most useful for the purposes of this research are: 1) the dimension of the column space of M, 2) the minimum number r of factors $u_i v_i^T$ such that $M = \sum_{i=1}^{r} u_i v_i^T$, and 3) the number of non-zero singular values of M. As a result of the second property, any rank r matrix can be decomposed as $M = UV^T$, where $U \in \mathbb{R}^{m \times r}, V \in \mathbb{R}^{n \times r}$. It can also be factored by the well-known singular value decomposition: $M = U \Sigma V^T$, where the columns of U and V are orthonormal bases of M's column- and row-space, respectively, and $\Sigma \in \mathbb{R}^{r \times r}$ is a diagonal matrix containing the singular values.

The rank function is non-convex (indeed it is piecewise constant), and thus can be problematic to optimize directly. Instead, it is often relaxed to the nuclear norm $\|M\|_*$ (also known as trace norm, Ky-Fan r-norm, or Schatten-1 norm), which is the sum of the singular values:

$$\|M\|_* = \sum_{i=1}^{r} \sigma_i. \tag{2.1}$$

The relaxation is simply the sparsity-inducing ℓ_1 norm applied to the vector of singular values, and is the tightest convex relaxation of the rank function over the set $\{X | \|X\| \leq 1\}$ [47]. Because the nuclear norm is convex, it has found widespread use in optimization problems that seek to minimize matrix rank. Some problems have conditions under which the optimal solution to a nuclear norm penalized problem can be proven to be identical to that of the rank constrained version [22, 23, 99]. In general, it has been demonstrated to be an effective heuristic for minimizing rank.

2.1.2 Tensors and Tensor Rank

Tensors are multi-dimensional arrays that generalize vectors and matrices. A vector is a first-order tensor; a matrix is a second-order tensor. The i-th mode refers to the i-th dimension of the tensor. The "rows" and "columns" of tensors are referred to generically as fibers; they are vectors obtained by fixing all but one dimension. A tensor $\mathcal{A}$ can be unfolded along the i-th mode into a matrix $A_{<i>}$: the mode-i unfolding simply arranges all of the mode-i fibers into a matrix. A matrix can then be folded back into a tensor; the exact ordering of fibers is not important so long as the same ordering is used during unfolding and folding. The mode-i product $\mathcal{T} = \mathcal{A} \times_i B$ is defined to be the folded version of $T_{<i>} = BA_{<i>}$, where $BA_{<i>}$ is matrix-matrix multiplication. The tensor outer product, denoted $\otimes$, generalizes the vector outer product. If a^j are vectors, the $i_1 i_2 \ldots i_n$ entry of $a^1 \otimes a^2 \otimes \cdots \otimes a^n$ is $a^1_{i_1} a^2_{i_2} \ldots a^n_{i_n}$.

The most straight-forward extension of rank to the tensor case defines the rank as the minimum number of rank-1 objects into which it can be decomposed. That is, the rank of a n-th-order tensor $\mathcal{T}$ is the smallest R for which there exist $\lambda \in \mathbb{R}^R, F^{(1)}, F^{(2)}, \ldots, F^{(d)} \in \mathbb{R}^{R\times n}$ such that

$$\mathcal{T} = \sum_{r=1}^{R} \lambda_r F^{(1)}_r \otimes F^{(2)}_r \otimes \cdots \otimes F^{(d)}_r. \tag{2.2}$$

This decomposition into factors is known as the CANDECOMP/PARAFAC (or CP) decomposition [69]. Unfortunately, not only is the tensor rank function non-convex, but even computing the rank for tensors greater than order 2 is NP-hard [53]. Another useful concept of tensor rank is the n-rank [69], which is a vector $\hat{R} = (R_1, R_2, \ldots, R_n)$. R_i is the dimension of the space spanned by the mode-i fibers. Put another way, R_i is the matrix rank of the i-th unfolding of the tensor. Researchers [112, 124] have proposed a convex nuclear norm for tensors that relaxes the tensor n-rank by applying the matrix nuclear norm to each of the tensor unfoldings.

2.1.3 Efficient SVD Computation

Most algorithms for matrix or tensor rank minimization require a singular value decomposition at each iteration. In general, the cost of computing the SVD of an $m \times n$ matrix with $m > n$ is $O(mn^2)$. It is of great practical importance to be able to compute this decomposition more efficiently. When only singular values above a given threshold μ are needed, one can compute a partial (truncated) SVD. This is true, for example, when computing a SVD that will subsequently be thresholded (as in equation 2.6). If there are r singular values above the threshold, the SVD computation requires only $O(mnr)$ operations. Packages such as PROPACK [71, 72] implement these operations efficiently. One can further improve efficiency with randomized algorithms [50, 43, 82], which are able to compute approximate SVDs in as little as $O(\max(m, n))$ time. Many of the randomized algorithms are designed to require only a constant number of passes over the data and require little memory. Brand [16] proposes a method that allows one to compute the SVD of the sum of a rank-r matrix and a rank-1 matrix in $O(r^3)$ time. Our research uses PROPACK to compute the SVD, but could in theory incorporate approximate or randomized SVDs to improve time complexity.

2.2 Prior Work in Rank Minimization and Matrix and Tensor Factorization

2.2.1 Example Rank Minimization Problems

Our research fits into a general class of optimization problems over matrices or tensors that either exactly or approximately constrain or penalize the rank. Four basic related forms are illustrated in Table 2.1. Ultimately, the goal is to estimate the optimal value θ out of the space Θ, where a penalty is incorporated on the rank of $H(\theta) \in \mathbb{X}$ (where $\mathbb{X}$ denotes either a

	Penalized	**Constrained**
Strict	$\min_{\theta\in\Theta} \quad g(\theta) + \mu\,\mathrm{rank}(H(\theta))$	$\min_{\theta\in\Theta} \quad g(\theta)$ s.t. $\quad \mathrm{rank}(H(\theta)) \leq r$
Relaxed	$\min_{\theta\in\Theta} \quad g(\theta) + \mu'\|H(\theta)\|_*$	$\min_{\theta\in\Theta} \quad g(\theta)$ s.t. $\quad \|H(\theta)\|_* \leq r'$

Table 2.1: Four basic rank minimization forms. $g : \Theta \to \mathbb{R}$. $h : \Theta \to \mathbb{X}$ and must not be constant.

Non-Convex Objective		Convex Relaxation	
Rank	$\mathrm{rank}(A)$	Nuclear norm	$\|A\|_*$
Cardinality	$\mathrm{cardinality}(A)$	ℓ_1 norm	$\|A\|_1$
Tensor n-Rank	$n\text{-rank}(\mathcal{A})$	Tensor nuclear norm	$\sum_i \|A_{<i>}\|_*$

Table 2.2: A summary of the convex relaxations used for a matrix A and tensor $\mathcal{A}$.

set of matrices or tensors). Arbitrary constraint sets can be encoded in the feasible set Θ. In Chapter 3, we see an example of the "Strict, Constrained" framework, which is a non-convex problem. In Chapters 4, 5, and 6, we see examples of the "Relaxed, Penalized" problem using the relaxations summarized in Tab. 2.2. These problems are convex but non-smooth, complicating optimization. In the following subsections we discuss several well known rank minimization problems that are special cases of this framework before discussing strategies for rank minimization itself.

Low rank matrix and tensor completion [19, 66, 21, 39, 22, 23, 40, 79, 119, 77, 48, 111]

In the matrix completion problem only a subset of the entries of a low rank matrix $M \in \mathbb{R}^{m \times n}$ are observed; namely, the set of entries M_{ij} for all (i, j) in an index set Ω. The goal is to recover the missing entries. The "parameters" $\theta \in \mathbb{R}^{m \times n}$ are simply the estimated entries of the matrix, and $H(\theta) = \theta$. If one assumes there is no noise in the observed entries, $g(\theta) = 0$ and the known entries are constrained by $\Theta = \{X \in \mathbb{R}^{m \times n} | X_{ij} = M_{ij}, \forall_{(i,j) \in \Omega}\}$. If bounded noise is permitted, the feasible set Θ can be expanded to include all matrices that deviate by no more than ϵ from the observed entries. Finally, one can simply penalize the deviations; e.g., letting $g(\theta) = \sum_{(i,j) \in \Omega} (\theta_{ij} - M_{ij})^2$. The tensor completion problem is solved analogously using the tensor nuclear norm.

Sparse plus low rank decomposition [24, 25, 89, 133, 20, 76]

Also known as "robust PCA," the goal of this problem is to decompose a given matrix $M \in \mathbb{R}^{m \times n}$ into the sum of a sparse matrix S and a low rank matrix L. Here, $\theta = [S, L]$, $g(\theta) = \|S\|_1$, $H(\theta) = L$, and $\Theta = \{[S, L] \in \mathbb{R}^{m \times 2n} | M = L + S\}$.

Min et al. [89] apply sparse and low-rank decomposition via the principal components pursuit algorithm [20] to the task of decomposing background topics from keywords. Specifically, they decompose a term-document matrix and find that, upon solving the robust PCA problem, the low rank component corresponds to a background model capturing topics, while the sparse component captures keywords - words of particular and unique significance to the given document. This decomposition of empirical counts is in contrast to the model parameter space decompositions we will see in Chapters 4 and 5.

Nuclear norm regularized least squares (NNLS) [123]

The goal of this problem is to find a low rank matrix $M \in \mathbb{R}^{m \times n}$ such that an affine mapping of M, $\mathscr{A}(M)$, is close to a vector $b \in \mathbb{R}^d$. Specifically, $\theta \in \mathbb{R}^{m \times n}$, $H(\theta) = \theta$, and $g(\theta) = \frac{1}{2}\|\mathscr{A}(\theta) - b\|_2^2$ for a given affine map $\mathscr{A} : \mathbb{R}^{n \times m} \to \mathbb{R}^d$.

2.2.2 Rank Minimization Algorithms

A number of algorithms have been introduced to solve matrix rank minimization problems of the forms of Table 2.1. Most solve specialized cases and exploit problem structure. Here we focus on generic approaches that can be applied to rank minimization.

One approach, first proposed in [46], is to reformulate the relaxed, penalized (nuclear norm) problem as a semidefinite program (SDP). With standard solvers, this does not scale beyond small (e.g. 100×100) matrices. Jaggi and Sulovský [59] propose a variant that does scale, using the approximate SDP solver of Hazan [51]. In their approach, each iteration is linear in the number of non-zero terms in the gradient of the function (at worst $O(nm)$), and it requires $O(1/\epsilon)$ iterations to obtain an ϵ-small primal-dual error. Another strategy [3] is to replace the non-smooth nuclear norm with a smooth approximation. One can then use gradient descent on this new objective. More recently, researchers [123, 61] have used sub-gradient descent to solve nuclear norm penalized problems, including accelerated versions that obtain an ϵ-accurate solution in $O(1/\sqrt{\epsilon})$ time. These solve the relaxed, penalized case of Table 2.1, which is rewritten below in terms of X.

$$\min_{X \in \mathbf{R}^{m \times n}} f(X) + \mu\|X\|_* \tag{2.3}$$

The algorithms we use in this thesis are variants of the accelerated proximal gradient algorithm of [123], which is listed in its original form in Algorithm 1. This algorithm makes use of the quadratic approximation $Q_\tau(X, Y)$ of $f(X) + \|X\|_*$ at Y:

$$Q(X, Y) = \frac{\tau^k}{2}\|X - G\|_F^2 + \mu\|X\|_* + f(Y) - \frac{1}{2\tau}\|\nabla f(Y)\|_F^2. \tag{2.4}$$

where $G = Y - \frac{1}{\tau}\nabla f(Y)$ for some parameter $\tau > 0$. Let $S_\tau(G)$ denote the minimizer of Q over X:

$$S_\tau(G) = \arg\min_X Q_\tau(X, Y). \tag{2.5}$$

There is in fact a closed form solution for $S_\tau(G)$. Let $G = U\Sigma V^T$ be the singular value decomposition, and let $D_\tau(\sigma) = \max(0, \sigma - \tau)$ be the thresholding operator. Then

$$S_\tau(G) = UD_{\mu/\tau}(\Sigma)V^T \tag{2.6}$$

where $D_\tau(\Sigma)$ denotes the element-wise application of $D_\tau(\sigma)$. One can also perform tensor nuclear norm minimization, which minimizes the tensor n-rank, using the convex multilinear estimation (CMLE) algorithm [112]. Gandy et al [48] also introduce a convex algorithm for identifying low n-rank tensors using the Douglas-Radford splitting technique and the alternating direction method of multipliers.

Algorithm 1: Accelerated Proximal Gradient Algorithm (from [123])

```
Input: A convex smooth function $f$
Choose $X^0 = X^{-1} \in \mathbb{R}^{m \times n}$, set $t^0 = t^{-1} = 1$. Set $k = 0$.
while not converged do
    Set $Y^k = X^k + \frac{t^{k-1}-1}{t^k}(X^k - X^{k-1})$
    Set $G^k = Y^k - \frac{1}{\tau^k}\nabla f(Y^k)$, where $\tau^k > 0$
    Set $S^k = S_{\tau^k}(G^k)$
    Choose stepsize $\alpha^k > 0$ and set $X^{k+1} = X^k + \alpha^k(S^k - X^k)$
    Set step size $t^{k+1} = \frac{1+\sqrt{1+4(t^k)^2}}{2}$
    Set $k = k + 1$
end
return $X$
```

2.2.3 *Multitask and Multiclass Learning*

In Chapter 1, we saw how low-rank bilinear and multilinear models could be seen as solving related problems simultaneously, and that this was a good fit for the language processing tasks we tackle in this thesis. Our approach was inspired by the multi-task learning literature. Multitask learning involves solving a set of related problems. By solving a set of related problems simultaneously, one can exploit overlap in the tasks to more robustly solve the individual tasks. Amit et al. [3] approach multiclass classification as a multitask problem. For each class $i \in \{1, \ldots, c\}$ they estimate a linear score function $s_i(x) = \theta_i^T x$, and apply the decision rule $f_\theta(x) = \arg\max_{i \in \{1,2,\ldots,c\}} \theta_i^T x$ to classify a point $x \in \mathbb{R}^d$. The individual weight vectors θ_c can be stacked into a weight matrix $\theta \in \mathbb{R}^{d \times c}$. Instead of applying the standard ℓ_2 penalty, $\|\theta\|_F$, they apply a nuclear norm penalty $\|\theta\|_*$. This has the effect of learning a low-dimensional representation of the data that is shared by all tasks. On two image recognition tasks they find that the model trained with nuclear norm regularization significantly outperforms that regularized with the Frobenius norm; this regularization is particularly useful when the number of training instances for the class is small. Argyriou et al. [4] consider a slightly different formulation where *sparse* shared representations are learned. Pong et al. [96] derive fast primal and dual accelerated proximal gradient algorithms for solving a nuclear norm regularized multitask learning problem.

2.2.4 Matrix Factorization

Although none of the methods we propose in this thesis fit into the category of matrix factorization directly, the concepts of low rank and factorization are closely linked. Given a low rank matrix (possibly estimated with a convex algorithm), one can convert it into factored form. Likewise, parameterizing with a factored matrix is a simple way to constrain the matrix rank. The primary disadvantage of using factorization to find low-rank matrices is that estimating a factored matrix is in general a non-convex problem, with the undesirable training implications that result. We sketch the literature on matrix factorization here for completeness.

A substantial amount of work has been conducted on matrix factorization methods, including their use in machine learning (e.g. [115]). Matrix factorization seeks to approximate $X \approx UV^T$ as a function of U and V under various constraints and criteria. Singh and Gordon [113] describe a unifying framework that encompasses a wide range of existing matrix factorization approaches, phrasing the problem as

$$\arg \min_{(U,V)\in\mathbb{C}} D(X\|f(UV^T), W) + R(U, V). \tag{2.7}$$

The function D is a generalized Bregman divergence [49], f is a link function mapping input to output, W is a matrix of weights on the penalties applied to entries, R is some regularizer and $\mathbb{C}$ imposes constraints on U and V. Into this framework they place the SVD, the weighted SVD [116], k-means clustering, k-medians clustering, probabilistic latent semantic indexing [54], non-negative matrix factorization [73], various forms of PCA [35, 104], and maximum margin matrix factorization [117]. Latent semantic analysis [42], being a SVD, is another obvious method of this type.

Matrix factorization can also be used to incorporate side information or otherwise transfer information between different domains through simultaneous (or "collective") factorization [113, 74]. For example, if one has two matrices that share a common dimension (e.g. a users-by-movie matrix X_1 and a movie-by-genre matrix X_2), the matrices can be simultaneously factored so that $X_1 \approx UV^T$ and $X_2 \approx VZ^T$. Solving these simultaneously forces V to learn a representation of users that is useful for both the movie and genre relationships. Due to the factored form used in matrix factorizations, these problems are non-convex and typically no guarantees can be made about the solution found.

2.2.5 Tensor Factorization

The advantage of tensors is that they allow more sophisticated modeling of multi-way interactions, which has been used in learning, for example, in [121]. The primary disadvantage of parameterizing with tensors is that they can easily have too many free parameters. By explicitly representing tensors in a factored low-rank form, one can significantly decrease the number of parameters.

Tensor factorizations have been used in a variety of application areas, including the adaptation of acoustic models for speech recognition [60], machine learning [70], approximate inference in graphical models [127], learning latent variable models [80, 98, 81], and phone recognition [38].

Much work has been done on the task of non-negative tensor factorization (NNTF). The connections between latent variable models and NNTF is addressed in [109], and the connection to tensorial probabilistic latent semantic analysis is described in [95]. We elaborate upon the relationship between low rank models and latent variables in Chapter 3 (where the model is interpreted as a mixture model). Some of the applications of NNTF include recommendation systems [34], speaker recognition [132], computer vision [110, 135], subject-verb-object co-occurrences [126], semantic word similarity [125] and music genre classification [9]. Without drawing the connection to low rank tensors, Lowd and Domingos [78] propose Naive Bayes models for estimating arbitrary probability distributions that can be seen as a generalization of the model introduced in Chapter 3. General algorithms for NNTF are presented in [134, 75, 136, 137]. One of these algorithms, based on expectation-maximization, is reviewed in Chapter 3.

2.3 Prior Work in Language Modeling

The primary application considered in this thesis is language modeling; here we review related work in n-gram model smoothing, maximum entropy language modeling and various continuous space language models.

2.3.1 n-gram Language Models and Smoothing

The dominant approach to language modeling uses a non-parametric n-gram approach combined with a method for smoothing the probability distributions. The fundamental idea in an n-gram language model is that the word sequence can be modeled as $(n-1)$th order Markov. That is, the probability of a sequence is assumed to decompose as follows

$$p(x_1, x_2, \dots, x_T) = \prod_{i=1}^{T} p(x_i | x_{i-n+1}, \dots, x_{i-1}) = p(x_i | h_i). \tag{2.8}$$

Here we let $h_i = x_{i-n+1}, \dots, x_{i-1}$ denote a *history*: the sequence of words that precede word x_i. Given a set of training sequences, the maximum likelihood estimate of the model is simply

$$p_{ML}(x|h) = \frac{C(hx)}{\sum_{x'} C(hx)} \tag{2.9}$$

where $C(z)$ denotes the count of n-gram z in the training data. Clearly, this probability will be zero for all h and x such that $C(hx) = 0$, with disastrous consequences for systems using the model (for example, a speech recognition system would be unable to recognize any sequence containing a zero probability n-gram sequence). It is crucial, therefore, that unobserved sequences not be assigned zero probability, but some very low non-zero probability. To accomplish this, researchers employ one of many methods for *smoothing* the probability distributions to ensure that $p(x|h) > 0$ for all x and h. Necessarily, this involves taking probability mass from events that have been seen and distributing them to unseen events, which "smooths" the distribution, hence the name. For a very thorough, if now slightly dated, comparison of smoothing methods, we refer the reader to [31]. To give an illustrative example of smoothing, we will describe modified Kneser-Ney smoothing, introduced in [31] and used throughout this thesis as a baseline for comparison. Empirical studies show that the modified Kneser-Ney method works well over a range of training set sizes on a variety of sources [31], although other methods are more effective when pruning is used in training large language models [26]. The modified Kneser-Ney (mKN) smoothed model, p_{mKN} is defined to be

$$\begin{aligned} p_{mKN}(x_i|x_{i-n+1},\ldots,x_{i-1}) \;&=\; \frac{C(x_{i-n+1},\ldots,x_i) - D(C(x_{i-n+1},\ldots,x_i))}{\sum_{x_i} C(x_{i-n+1},\ldots,x_i)} \\ &\quad + \gamma(x_{i-n+1},\ldots,x_{i-1}) p_{mKN}(x_i|x_{i-n+2},\ldots,x_{i-1}) \end{aligned} \tag{2.10}$$

where D serves to discount the maximum likelihood counts C, and is a function of $C(x_{i-n+1},\ldots,x_i)$ as follows:

$$D_1 = 1 - 2Y\frac{n_2}{n_1}, \quad D_2 = 2 - 3Y\frac{n_3}{n_2}, \quad D_{3+} = 3 - 4Y\frac{n_4}{n_3}, \quad Y = \frac{n_1}{n_1 + 2n_2}. \tag{2.11}$$

That is, the counts of n-grams that we observe only once in the training data will be discounted by D_1; those that were observed twice discounted by D_2, and those observed three or more times discounted by D_{3+} (where n_i is the total number of n-grams with count equal to i). So the first term in Eqn. 2.10 is analogous to the maximum likelihood estimate, with some adjustments to the counts. The second term *interpolates* the n-gram estimate with an $(n-1)$-gram mKN model. γ is chosen to ensure that the overall probability distribution sums to one; its details are given in [31]. The idea of interpolation with lower order models is common in language model smoothing; it gives a simple but relatively effective way of balancing richer models (higher order n) that are difficult to estimate but potentially more predictive with simpler models (lower order n) that can be better estimated, at the expense of some predictive power.

Another popular way to smooth n-gram language models is with a class-based model [17]. Here we assume that we have a function that maps a word x_i deterministically to its corresponding word class c_i (for example, its part-of-speech class). In a class-based model, one can decompose the conditional word probability as

$$p_C(x_i|x_{i-n+1},\ldots,x_{i-1}) = p(x_i|c_i)p(c_i|c_{i-n+1},\ldots,c_{i-1}). \tag{2.12}$$

Because the conditional probability tables defining the two conditional probabilities on the right side of Eqn. 2.12 are smaller, they can be much more robustly estimated from the training data.

Whereas class-based language models associate a class with each word, the factored language model [12] associates a set of classes, called *factors*, with each word. Designed for use with morphologically rich languages, the factors of an Arabic word might include the stem, the root and the morphological pattern. A word can be written as the set of its factors; for example,e $x_i = \{f_i^1 f_i^2 f_i^3\}$. Then the conditional probability $p(x_i|x_{i-n+1},\ldots,x_{i-1})$ can be written in terms of the factors as

$$p(x_i|x_{i-n+1},\ldots,x_{i-1}) = p(f_i^1 f_i^2 f_i^3|f_{i-n+1}^1, f_{i-n+1}^2, f_{i-n+1}^3, \ldots, f_{i-1}^1, f_{i-1}^2, f_{i-1}^3). \tag{2.13}$$

Using factors instead of a single class means that the factored language model has more degrees of freedom, which Bilmes and Kirchhoff take advantage of using the idea of *generalized backoff*. The idea is straightforward: assuming there are not enough instances of a given history-word n-gram to robustly estimate the conditional probability, one can drop some of the factors on which Eqn. 2.13 is conditioned. For example, even if one had not observed a sequence of two Arabic words frequently enough to trust the estimate, one might trust the estimate of the second word following the stem of the first word. Bilmes and Kirchhoff provide numerous strategies for generalized backoff; refer to [12] for details. Gains from factored language models have been reported in subsequent work [128, 44].

2.3.2 Maximum Entropy Models

Maximum entropy models, also known as exponential or log-linear models, are popular in language processing. In a maximum entropy language model [101], the relationship between a word x and its history h is encoded through a vector-valued feature function $f(x, h)$. Given f, the conditional probability $p(x|h)$ is defined to be

$$p(x|h) = \frac{\exp(a^T f(x, h))}{\sum_{x'} \exp(a^T f(x', h))}. \tag{2.14}$$

The features $f(x, h)$ are typically an n-gram indicator vector, but one can also use features of word classes or long range dependencies ("triggers"). Learning involves estimating the weight vector $a \in \mathbb{R}^m$ that maximizes the data log-likelihood, which in turn corresponds to finding the maximum entropy distribution such that the feature expectations under the model and empirical distributions match (hence the name).

Chen found that the ℓ_1 norm of the weight vector a was closely tied to model generalization performance [28]; specifically that the average log-likelihood of the test set is approximately equal to the average log-likelihood of the training set plus a scaled ℓ_1 norm of a. Motivated by this empirical evidence in favor of models that fit the training data well with less overall weight mass, he introduced a new class-based model, termed Model M [29]. It decomposes the probabilities as follows:

$$p(w_1, \ldots, w_T) = \left(\prod_{j=1}^{T+1} p(c_j | c_1, \ldots, c_{j-1}, w_1, \ldots, w_{j-1}) \right) \left(\prod_{j=1}^{T} p(w_j | c_1, \ldots, c_j, w_1, \ldots, w_{j-1}) \right) \tag{2.15}$$

Here c_i denotes the class label of word i and c_{T+1} is a end-of-sentence token. Model M uses exponential models for both conditional probability distributions and obtains significant gains in perplexity over baseline n-gram models that use Katz and modified Kneser-Ney smoothing. As a bonus, introducing the classes means that the normalization factors of each model sum over a smaller number of items, providing a speed advantage. Subsequent work has improved the scalability of the model [32, 33, 107], improved word classing strategies [30, 45] and incorporated new features [138].

2.3.3 Continuous Representations for Language Modeling

There are many potential advantages to representing discrete objects with continuous representations, including natural notions of distance and other geometric manipulations of the objects. Mapping to a sufficiently low-dimensional space can also help to compensate for training data sparsity. A prototypical example of using continuous representations of words is latent semantic analysis [42], where words are embedded into a lower dimensional space defined by the SVD of a word-by-document co-occurrence matrix. In that application, distances in the continuous space are intended to capture the notion of semantic similarity.

Continuous space models have also been advocated and pursued among many in the field of language modeling, where the representations learned encode primarily syntactic, but also potentially topical and stylistic, similarity. Perhaps the best known example of these are neural network language models [11, 106], where first a discrete representation of the history (usually the concatenation of word indicator vectors) is mapped to a continuous representation of the history (usually a concatenation of low dimensional continuous word representations), and then the continuous history representation is fed into a single hidden layer neural network that predicts the following word. Both the continuous low-dimensional word representations and the neural network parameters are learned from the data. Recently, neural network language models have been extended to consider deep networks (with multiple hidden layers) [6] and recurrent neural networks [86, 84, 83, 87]. The latter, in particular, shows very good improvements in perplexity by taking advantage of an arbitrarily long history context, although only a finite context is practical for speech recognition decoding or lattice rescoring. Though effective, neural network language models pose some challenges for training in terms of both time complexity and sensitivity to parameters.

Another example of continuous representation language models are latent variable and factored models [94, 93, 90, 91, 92, 15]. Son et al. [114] offer a comparison of and some practical observations on different continuous space language model approaches, including neural network approaches. One of their observations is that local optima do occur and must be dealt with; this will not be the case for many of the models we propose.

As is shown in Chapter 4, one may also incorporate a feature-rich representation of words. Then the mapping is not from discrete word to continuous space, but from feature representation to continuous space. A feature-based representation offers the same advantages as factors did above in the Factored Language Model. In a follow up study to the original factored language model work, Alexandrescu and Kirchhoff [2] introduced neural factored language models, which feed a factored representation into a neural language model.

Mnih proposed several low rank tensor factorization approaches to language modeling, with the goal of capturing interactions between continuous space representations of words [90]. A matrix R maps words into a continuous representation. Each of the history vectors for a fixed history length are multiplied by a position-dependent "interaction" matrix and summed. The inner-product between the aggregated history and the representation of the predicted word is used as the score. Some additional factored forms are considered in [92] that allow a restricted style of non-linear interaction between history words. The mapping and interaction matrices are iteratively learned in a non-convex training procedure. In contrast to Mnih's models, the methods we will introduce in Chapter 4 learn the dimensionality of the representation at the same time as the representation is learned, and use a convex training objective to guarantee optimality of the solution.

Chapter 3

THE LOW RANK LANGUAGE MODEL

Language model smoothing has been well studied, and it is widely known that performance improves substantially when training on large data sets. While large training sets are valuable, there are situations where they are not available, including system prototyping for a new domain or training language models that specialize for communicative goals or roles. Existing, non-parametric language model smoothing methods control for complexity primarily through the n-gram order and through thresholds on the minimum number of observations needed for a specific parameter estimate. This thesis provides alternative approaches based on the concepts of rank and sparsity in matrices and tensors. In this chapter, we cast the smoothing problem as low rank tensor estimation.[1] By permitting finer control over model complexity, our low rank language models are able to fit the small in-domain data with better generalization performance.

3.1 Background

3.1.1 Notation

Let R denote the tensor rank, n the order of the n-gram model, V the size of the vocabulary, and K the number of n-gram tokens in the training set. Let $\mathbb{R}^p$ denote the set of vectors of length p, and $\mathbb{R}^{p\times q}$ the set of $p \times q$ matrices. Tensors are written in script font (e.g. $\mathcal{T}$).

3.1.2 Rank in Language Modeling

Every n-gram language model implicitly defines an nth-order joint probability tensor $\mathcal{T}$:

$$P(w_1 w_2 \dots w_n = i_1 i_2 \dots i_n) = \mathcal{T}_{i_1 i_2 \dots i_n}. \tag{3.1}$$

[1]Portions of this chapter appeared previously in [58].

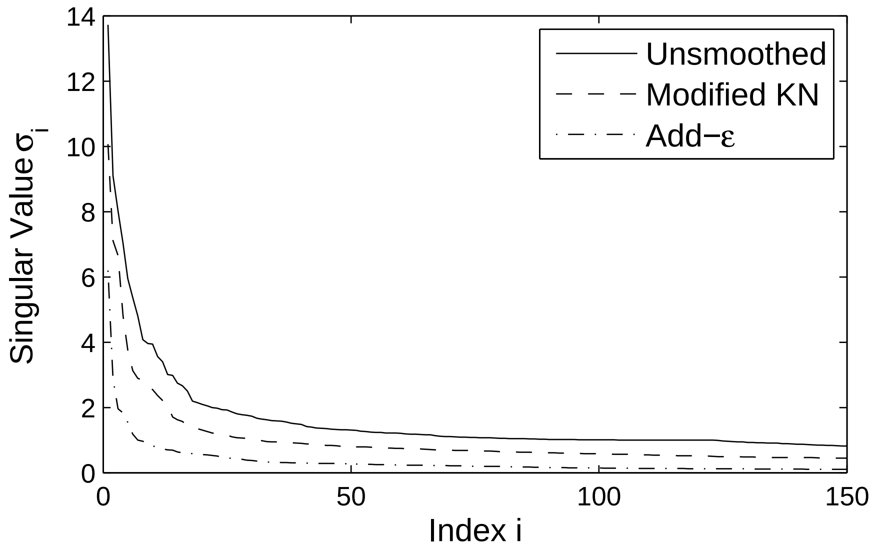


Figure 3.1: Singular values of conditional probability matrix for unsmoothed (solid blue), modified-KN smoothed (dashed red), and add-ϵ smoothed (dash-dotted black) models. Trained on 150K words of broadcast news text, with a 5K word vocabulary.

An unsmoothed maximum likelihood-estimated language model can be viewed as an entry-wise sparse tensor. The obvious problem with parameterizing a language model directly by the entries of $\mathcal{T}$ is that, under nearly all conditions, there are too many degrees of freedom for reliable estimation. Hence, a substantial amount of research has gone into the smoothing of n-gram language models.

One can compare different smoothing methods by their effects on the properties of $\mathcal{T}$. In particular, even highly distinct approaches to smoothing have the effect of reducing, either exactly or approximately, the rank of the tensor $\mathcal{T}$. Reducing the rank implies a reduction in model complexity, yielding a model that is easier to estimate from the finite amount of training data. In the matrix case, reducing the rank of the joint probability matrix is equivalent to pushing the distributions over a vocabulary of size V, $P(\cdot|w_{t-1})$, either exactly or approximately into a subspace of $\mathbb{R}^V$. More generally, a low rank tensor implies that the set of distributions $P(\cdot|\cdot)$ are largely governed by a common set of $R \ll V$ underlying factors. Although the factors need not be interpretable, and certainly not pre-defined, one might envision that a set of syntactic (e.g. part-of-speech), style and/or semantic factors could account for much of the observed sequence behavior in natural language. Fig. 3.1 illustrates the rank-reducing phenomenon of smoothing in a conditional probability matrix. Both modified Kneser-Ney and add-ϵ [31] smoothing shrink the mass of the singular values over the unsmoothed estimate. This effect is most pronounced on small training sets, which require more smoothing.

The number of factors (the rank R) of a tensor thus provides a mechanism to control model complexity. The benefits of controlling model complexity are well-known: a model that is too expressive can overfit the training, while a model that is too simple may not be able to capture the inherent structure. By reducing the mass of singular values, existing smoothing methods effectively reduce the complexity of the model. Although they do so in a meaningful and interpretable way, it is unlikely that any fixed approach to smoothing will be *optimal*, in the sense that it may return a model whose complexity is somewhat more or somewhat less than ideal for the given training data. In this chapter we test the hypothesis that it is the rank-reducing behavior that is important in the generalization of smoothed language models, and propose a model and training approach better matched to this objective.

3.1.3 Tensor Rank Minimization

There are two dominant approaches to estimating low rank tensors. One approach solves an optimization problem that penalizes the tensor rank, which encourages but does not impose a low rank solution:

$$\min_{\mathcal{T}\in\mathbb{T}} \quad f(\mathcal{T}) + \operatorname{rank}(\mathcal{T}). \tag{3.2}$$

($\mathbb{T}$ denotes the feasible set.) Recall from Chapter 2 that the rank of a tensor $\mathcal{T}$ is defined to be the minimum R such that it can be written as

$$\mathcal{T} = \sum_{r=1}^{R} \lambda_r F_r^{(1)} \otimes F_r^{(2)} \otimes \cdots \otimes F_r^{(d)}. \tag{3.3}$$

where $F_r^{(i)}$ is a vector and $\otimes$ denotes the tensor outer product. When the tensor is order-3 or higher, not only is this problem NP-hard [53], but there are no tractable convex relaxations of this notion of rank. Instead, one can impose a hard rank constraint:

$$\min_{\mathcal{T}\in\mathbb{T},\ \mathrm{rank}(\mathcal{T})\leq R} \quad f(\mathcal{T}). \tag{3.4}$$

In this non-convex problem, R is a pre-determined hard limit; in practice, the problem is solved repeatedly for different R and the best result is used. This approach allows one to reduce the space complexity to $O(nRV)$ by explicitly encoding the parameters in the low-rank factored form of Eqn. 3.3, which makes scaling to real-world datasets practical.

3.2 Low Rank Language Models

3.2.1 Model

Our low rank language models (LRLMs) represent n-gram probabilities in a factored tensor form:

$$\begin{aligned} P(w_{t-n+1},\ldots,w_t = i_1 i_2 \ldots i_n) &= \mathcal{T}_{i_1 i_2 \ldots i_n} \\ &= \sum_{r=1}^{R} \lambda_r F^{(1)}_{r i_1} F^{(2)}_{r i_2} \ldots F^{(n)}_{r i_n}. \end{aligned} \tag{3.5}$$

The model is parametrized by the non-negative component weights $\lambda \in \mathbb{R}^R$ and the factor matrices $F^{(i)} \in \mathbb{R}^{R\times V}$.

Because $\mathcal{T}$ denotes a joint probability distribution, we must impose that $\mathcal{T}$ is entry-wise non-negative and sums to one. If all of the parameters in Eqn. 3.5 are non-negative, we can constrain the rows of $F^{(i)}$ to sum to one. It is then sufficient to constrain λ to sum to one for $\mathcal{T}$ to sum to one. We will see later that requiring our parameters to be non-negative provides substantial benefits for interpretability and leads to an efficient training algorithm. Technically, R denotes the *non-negative tensor rank*, which is never less than the tensor rank. Under these constraints, the rows of the factor matrices can be interpreted as position-dependent unigram models over our vocabulary, and the elements of λ as priors on each component:

$$\begin{aligned} P(w_{t-n+1},\ldots,w_t = i_1 \ldots i_n) &= \sum_{r=1}^{R} \lambda_r F^{(1)}_{r i_1} F^{(2)}_{r i_2} \ldots F^{(n)}_{r i_n} \\ &= \sum_{r=1}^{R} P(r) P^{(1)}_r(w_{t-n+1}) \ldots P^{(n)}_r(w_t). \end{aligned} \tag{3.6}$$

Note that when $R = 1$, $\mathcal{T}$ degenerates to a unigram model. On the other extreme, when $\mathcal{T}$ is sufficiently high rank ($R = V^{n-1}$), it can represent any possible joint probability distribution over n words. Interpolating R between these extremes permits us to carefully control model complexity, so that it can be matched to the amount of training data available.

We construct the probability of a word sequence using the standard n-gram Markov assumption:

$$\begin{aligned} P(w_1, \ldots, w_T) &= \prod_{t=1}^{T} P(w_t | w_{t-n+1}^{t-1}) \\ &= \prod_{t=1}^{T} \frac{\sum_{r=1}^{R} P(r) P_r^{(1)}(w_{t-n+1}) \cdots P_r^{(n)}(w_t)}{\sum_{s=1}^{R} P(s) P_s^{(1)}(w_{t-n+1}) \cdots P_s^{(n-1)}(w_{t-1})}, \end{aligned} \tag{3.7}$$

where for notational convenience we assume that $w_{1-n+1}, \ldots, w_0$ are a designated sentence start token. Note that in Eqn. 3.7, unlike traditional language mixture models, $P(w_t | w_{t-n+1}^{t-1})$ does not take the form of a sum of conditional distributions. By learning joint probabilities directly, we can capture higher-order multilinear behavior.

3.2.2 Training

Our criterion for language model training is to maximize the log-likelihood of the n-grams appearing in the training data. Formally, we find a local solution to the problem:

$$\max_{\mathcal{T} \in \mathbb{P},\ \mathrm{rank}(\mathcal{T}) \leq R} \log P_{\mathcal{T}}(D), \tag{3.8}$$

where $\mathbb{P}$ denotes the set of element-wise non-negative tensors whose entries sum to one, i.e. the set of tensors corresponding to valid joint probability distributions; $D = \{d_1, d_2, \ldots, d_K\}$ are the n-grams in the training data (obtained by sliding a window of size n over each sentence); and $P_{\mathcal{T}}$ is the probability distribution given by $\mathcal{T}$.[2] For traditional n-gram models, the maximum likelihood objective yields models that are highly overfit to the data; in particular, they are plagued with zero probability n-grams. The parameter tying implied

[2]By using overlapping n-grams, the samples are no longer independent, and the estimation is not strictly maximum likelihood of the original data. While no token is double counted in a distribution $P_r^{(i)}$, each will be counted in $P_r^{(i)}$ for multiple i. The implication is that the distribution is not consistent with respect to marginalization; e.g., the probability of the start symbol is position-dependent (a desirable property).

by the low rank form greatly reduces the risk of introducing zero probabilities into the model; in practice, some additional smoothing is still required.

The low-rank language model can be interpreted as a mixture model, where each component is a joint distribution that decomposes into a product of position-dependent unigram models over words: $P_r(w_{t-n+1}, \ldots, w_t) = P_r^{(1)}(w_{t-n+1}) \cdots P_r^{(n)}(w_t)$. Using this interpretation, we propose an expectation-maximization (EM) approach to training our models, iterating:

1. Given model parameters, assign the responsibilities γ_{rk} of each component r to the k-th n-gram instance $d_k = (w_1^{(k)}, w_2^{(k)}, \ldots, w_n^{(k)})$:

$$\gamma_{rk} = \frac{P(d_k|r)P(r)}{P(d_k)} = \frac{P(r)P_r^{(1)}(w_1^{(k)}) \cdots P_r^{(n)}(w_n^{(k)})}{P(w_1^{(k)}, w_2^{(k)}, \ldots, w_n^{(k)})} \tag{3.9}$$

2. Given responsibilities γ, re-estimate $F^{(i)}, \lambda$:

$$P_r^{(p)}(w) = \frac{\sum_{k=1}^{K} \gamma_{rk}\delta(w_p^{(k)} = w)}{\sum_{k=1}^{K} \gamma_{rk}} \tag{3.10}$$

$$P(r) = \frac{1}{K}\sum_{k=1}^{K} \gamma_{rk} \tag{3.11}$$

where δ is an indicator function. Iterations continue until perplexity on a held-out development set begins to increase.

The above training is only guaranteed to converge to a local optimum, which means that proper initialization can be important. A simple initialization is reasonably effective for bigrams: randomly assign each training sample to one of the R mixture components and estimate the component statistics similar to step 2. To avoid zeroes in the component models, a small count mass weighted by the global unigram probability is added to each distribution $P_r^{(p)}(w)$ in Eqn. 3.10.

3.3 English Genre Modeling Experiments

Our expectation is that the LRLM will be good for applications with small training sets. The experiments here first evaluate the LRLM by training on a small set of conversational

Dataset	Size (words)
BN Train	3.2M
BC Train	99K
BC Dev	189K
BC Test	136K

Table 3.1: Language model experiment data.

speech transcripts and then in a domain adaptation context, which is another common approach when there is data sparsity in the target domain. The adaptation strategy is the standard approach of static mixture modeling, specifically linearly interpolating a large general model trained on out-of-domain data with the small domain-specific model.

3.3.1 Experimental Setup

Our experiments use LDC English broadcast speech data,[3] with broadcast conversations (BC) or talkshows as the target domain. This in-domain data is divided into three sets: training, development and test. For the out-of-domain data we use a much larger set of broadcast news speech, which is more formal in style and less conversational. Table 3.1 summarizes the data sets.

We train several bigram low rank language models (LR2) on the in-domain (BC) data, tuning the rank (in the range of 25 to 300). Because the initialization is randomized, we train models for each rank 10 times with different initializations and pick the one that gives the best performance on the development set. As baselines, we also train in-domain bigram (B2) and trigram (B3) standard language models with modified Kneser-Ney (mKN) smoothing. Our general trigram (G3), trained on BN, also uses mKN smoothing. Finally, each of the in-domain models is interpolated with the general model. We use the SRILM toolkit [118] to train the mKN models and to perform model interpolation. The vocabulary consists of the top 5K words in the in-domain (BC) training set.

[3]http://www.ldc.upenn.edu

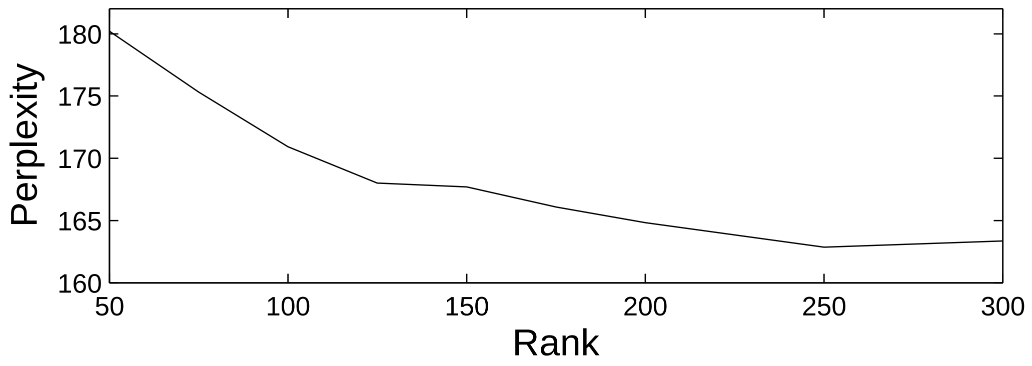


Figure 3.2: Low rank language model perplexity by rank.

3.3.2 Results

The experimental results are presented in Table 3.2. As expected, models using only the small in-domain training data have relatively high perplexities. Of the in-domain-only models, however, the LRLM gives the best perplexity, 3.6% lower than the best baseline. Notably, the LR bigram outperforms the mKN trigram. The LR trigram gave no further gain; extensions to address this are described later. The LRLM results are similar to mKN when training on the larger BN set.

Benefiting from a larger training set, the out-of-domain model alone is much better than the small in-domain models. Interpolating the general model with any of the in-domain models yields an approximately 15% reduction in perplexity over the general model alone, highlighting the importance of in-domain data. However, the different target-domain models are contributing complementary information: when the in-domain models are combined performance further improves. In particular, combining the baseline trigram and LRLM gives the largest relative reduction in perplexity.

Figure 3.2 reports LRLM perplexity for the LR2 model by rank R (the number of mixture components). For an in-domain bigram model, using approximately $R = 250$ mixture components is optimal, which corresponds to roughly 10% as many parameters as a full bigram joint probability matrix.

Model	Perplexity
B2	166.7
B3	169.1
LR2	**162.9**
B2+LR2	154.5
G3	98.7
G3+B2	83.7
G3+B3	83.6
G3+LR2	83.6
G3+B2+LR2	83.1
G3+B3+LR2	**82.6**

Table 3.2: In-domain test set perplexities. B denotes in-domain baseline model, G denotes general model, and LR denotes in-domain low rank model. Each model is suffixed by its n-gram order.

3.3.3 Discussion

Each component in the model specializes in some particular language behavior; in this light, the LRLM is a type of mixture of experts. To gain insight into what the different LRLM components capture, we investigated likely n-grams for different mixture components. We find that components tend to specialize in one of four ways: 1) modeling the distribution of words following a common word, 2) modeling the distribution of words preceding a common word, 3) modeling sets of n-grams where the words in both positions are relatively interchangeable with the other words in the same position, and 4) modeling semantic related n-grams. Table 3.3 illustrates these four types, showing sample n-grams randomly drawn from different components of a trained low rank model.

r = 53	r = 33	r = 236	r = 122
λ_r = 1.29e-02	λ_r = 1.41e-02	λ_r = 1.26e-02	λ_r = 1.26e-03
he was	down and	should be	we civilians
he goes	over and	can be	defense security
he says	people and	will make	syrian armed
he faces	one and	would affect	iraqi prison

Table 3.3: Samples drawn randomly from LRLM mixture components.

3.4 Conclusions

Language model smoothing techniques can be viewed as operations on joint probability tensors over words; in this space, it is observed that one common thread between many smoothing methods is to reduce, either exactly or approximately, the tensor rank. This chapter introduces a new approach to language modeling that more directly optimizes the low rank objective, using a factored low-rank tensor representation of the joint probability distribution. Using a novel approach to parameter-tying, the LRLM is better suited to modeling domains where training resources are scarce. On a genre-adaptation task, the LRLM obtains lower perplexity than the baseline (modified Kneser-Ney-smoothed) models.

Our initial experiments did not obtain gains for trigrams as for bigrams. Possible improvements that may address this include alternative initialization methods to find better local optima (since training optimizes a non-convex objective), exploration of smoothing in combination with regularization, and other low-rank parameterizations of the model (e.g. the Tucker decomposition [69]). For domain adaptation, there are many other approaches that could be leveraged [8], and the LRLM might be useful as the filtering LM used in selecting data from out-of-domain sources [18]. Finally, it would be possible to incorporate additional criteria into the LRLM training objective, e.g. minimizing distance to a reference distribution.

Chapter 4

THE SPARSE PLUS LOW-RANK LANGUAGE MODEL

In Chapter 3 we introduced a tensor-based “low rank language model” (LRLM) that outperforms baseline models when training data is limited. A disadvantage of the LRLM is that the non-convex objective complicates training. In this chapter, we propose a new model based on a reparameterization of the maximum entropy modeling (exponential) framework, side-stepping the limitations of the LRLM and leveraging advantages of exponential models.[1] Like existing exponential models, our model can make use of features of words and histories and training is convex. Unlike existing models, however, we parameterize the model with the sum of two weight matrices: a low rank matrix that effectively models frequent, productive sequential patterns, and a sparse matrix that captures exceptional sequences. We will show that the low rank weight matrix can be interpreted as incorporating a continuous-space language model into the exponential setting, and will discuss how the sparsity pattern in the sparse weight matrix can benefit other, important language processing tasks.

We evaluate the model and its properties in a sequence of experiments. First we evaluate its language modeling performance on English telephone conversations, which allows us to gain insight into the model’s behavior. We then evaluate the how well the model does with low resource languages, reporting results using limited training data from Cantonese, Pashto, Turkish, Tagalog and Vietnamese. To explore the model’s ability to incorporate non-trivial features of words and histories, we investigate morphological features in further Turkish language modeling experiments. Finally, we conclude with an analysis of how we can use learned sparse weights to automatically discover words and multiwords in Cantonese and Vietnamese.

[1]Portions of this chapter appeared previously in [56].

4.1 Sparse and Low-Rank Language Models

4.1.1 The Model

As described in Chapter 2, the standard exponential language model [101] has the form

$$p(w|h) = \frac{\exp\left(a^T f(w,h)\right)}{\sum_{w'} \exp\left(a^T f(w',h)\right)}, \tag{4.1}$$

where $a \in \mathbb{R}^d$ are the parameters and $f(w,h) \in \mathbb{R}^d$ is the feature vector extracted from word w and history h. We generalize the model with two key changes: i) the vector is recast as a feature matrix with a corresponding weight matrix A, and ii) the weight matrix A is decomposed into the sum of two matrices, $A = S + L$, each of which have special structure. The basic weight matrix parameterized exponential model can be written as:

$$p(w|h) = \frac{\exp\left(\psi(w)^T A \phi(h)\right)}{\sum_{w'} \exp\left(\psi(w')^T A \phi(h)\right)}. \tag{4.2}$$

Here $\psi(w) \in \mathbb{R}^{d_\psi}$ is the vector of features of w individually and $\phi(h) \in \mathbb{R}^{d_\phi}$ are the features of h. In this case $A \in \mathbb{R}^{d_\psi \times d_\phi}$ is a parameter *matrix*. This can be linked to the notation of Eqn. 4.1 by noting that $\psi(w)^T A \phi(h) = \langle A, \psi(w)\phi(h)^T \rangle$, where $\langle \cdot, \cdot \rangle$ denotes a matrix inner-product (element-wise multiply and sum). If one denotes the vectorization of A with a and the vectorization of feature matrix $\psi(w)\phi(h)^T$ with $f(w,h)$, then we can convert any model of the form Eqn. 4.2 into one of the form of Eqn. 4.1. On the other hand, any exponential language model of the form Eqn. 4.1 *whose features are products of features on words and histories* can transformed into one in the form of Eqn. 4.2. In particular, standard n-gram features take this form, where $\psi(w)$ and $\phi(h)$ are one-hot (indicator) encodings of the words and histories. In most of our experiments we focus on these standard n-gram features, though we branch out to morphological features in Sec. 4.5.

Empirically we observe that the weight matrices A learned for models of the form Eqn. 4.2 contain two qualitatively different kinds of information. First, there are relatively dense regions of the matrix that model the sequential behavior of high frequency words. Because only a small fraction of the words are frequent, this information can be well modeled by a low rank matrix. Second, there are large sparse regions of the matrix, where only a handful of the elements deviate significantly from zero - these correspond to n-grams whose individual

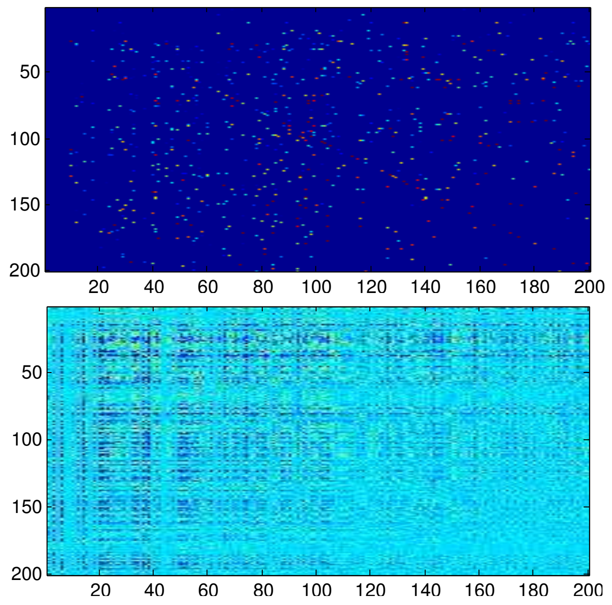


Figure 4.1: The weight matrix A can be naturally decomposed into sparse (top) and low rank (bottom) matrices.

words infrequently appear outside of a small handful of n-grams (e.g. "san francisco"). This information alone can be well modeled by a sparse matrix. Fig. 4.1 illustrates this result, visualizing the estimated weights in the 200×200 leading submatrix of a bigram weight matrix trained on 100K tokens with a 5K vocabulary. The complete matrix can be decomposed accurately into the sum of a sparse matrix (S, upper) and low rank matrix (L, lower). We show in this chapter that language modeling performance can be improved by using a more general model able to efficiently capture both types of structure inherent in the data.

Sparse Component

The model of Eqn. 4.1 is often trained with ℓ_1 regularization applied to a. Not only is it well-known that this particular penalty leads to *sparse* solutions, but it empirically has also been found to be a good criterion to minimize (in terms of test set perplexity) [28]. An entry-wise ℓ_1 penalty can be applied to a weight matrix S to the same effect. In standard models, the "sparse" component is the only component, and is thus tasked with modeling all of the sequential behavior; we will see below how our model uses sparsity in a different way.

Low-Rank Component

Restricting ourselves to a sparse solution ignores an important attribute of language: that similarities exist between words and between histories in the data. A sparse model has no way to exploit similarities that might exist (e.g. between the words "bicycle" and "bike"). Viewed in the form of Eqn. 4.1, this is inevitable: features are values at arbitrary positions in a vector. Viewed in the form of Eqn. 4.2, we see similarities between two words can be expressed by similarity between the corresponding rows, and similarities between histories can be viewed as similarities between the corresponding columns. More generally, sets of rows (or columns) may live in subspaces, e.g. one might envision a "space" of adjectives, or a "space" of nouns. This property corresponds to a *low rank* solution; i.e. finding a low rank weight matrix A.

Empirically, a low rank component typically appears in the solution without any encouragement from the model or training. By facilitating the existence of a low-rank component, we can improve the modeling performance. For example, the co-occurrence statistics for a word α that has been observed 50 times may be sufficiently similar to a set of words β that have been observed hundreds or thousands of times for α's weights to be pushed into β's subspace of weights; in effect, this "fills in" missing entries from α's weight rows and columns.[2]

The idea of learning and exploiting similarities between objects (e.g. words and histories) is a common theme in the literature on learning shared representations [3] and is used by language models with continuous representations of words, particularly neural network language models [10, 106]. We show here that an exponential model with a low rank weight matrix A is in fact a continuous-space language model. To see this, note that $A \in \mathbb{R}^{d_\psi \times d_\phi}$ with rank $R \leq \min(d_\psi, d_\phi)$ has a singular value decomposition $A = U\Sigma V^T$, with diagonal

[2]This basic idea is extensively exploited in low-rank matrix completion approaches to collaborative filtering, as discussed in Chapter 2.

matrix of singular values $\Sigma \in \mathbb{R}^{R\times R}$, left singular vectors $U \in \mathbb{R}^{d_\psi \times R}$ and right singular vectors $V \in \mathbb{R}^{d_\phi \times R}$. Substituting the structure of A into Eqn. 4.2, we get

$$\begin{aligned} p(w|h) &= \frac{1}{Z(h)} \exp\left(\psi(w)^T U \Sigma V^T \phi(h)\right) && (4.3) \\ &= \frac{1}{Z(h)} \exp\left(\left(U^T \psi(w)\right) \Sigma \left(V^T \phi(h)\right)\right) && (4.4) \\ &= \frac{1}{Z(h)} \exp\left(\tilde{\psi}(w)^T \Sigma \tilde{\phi}(h)\right). && (4.5) \end{aligned}$$

Here $\tilde{\psi}(w) = U^T \psi(w)$ denotes a continuous, low-dimensional representation of w, $\tilde{\phi}(w) = V^T \phi(h)$ denotes a continuous, low-dimensional representation of h, and $Z(h)$ is the normalizing factor. The probability of a word following a history is proportional to the weighted inner-product $\tilde{\psi}(w)^T \Sigma \tilde{\phi}(h)$ in the low-dimensional space. A illustrative example is given in Fig. 4.2, which plots the hypothetical continuous representation of three different histories and three different prediction position words; the inner products induced by this embedding means that "next Tuesday" and "next Thursday" (with a positive inner product) are probable sequences, while "orange Tuesday" and "orange Thursday" (with a negative inner product) are not. The dimension of the continuous representation is equal to R, the rank; lower rank solutions for A correspond to embeddings of words in lower dimensional spaces. Similar words will have continuous representations that are close to each other in the low-dimensional space; the same is true for similar histories. Crucially, the low dimensional representation of a word can (and should) be different in the history position h than in the predictive position w. In Section 4.1.2, we present an algorithm that discriminatively learns a low rank matrix, and thus discriminatively learns low dimensional continuous representations of words and histories.

When A is low rank, the model of Eqn. 4.2 bears some similarity to Mnih and Hinton's "log-bilinear" language model [90], which estimates a matrix that is analogous to our U in Eqn. 4.4, and matrices analogous to V for each word in a fixed history window. In doing so, they find continuous low-dimensional representations of words and history words. A few advantages of our approach are that 1) we are guaranteed to converge to a globally optimal solution, 2) we support arbitrary feature functions of words and histories, and 3) the dimension of the hidden representation is learned, rather than pre-specified.

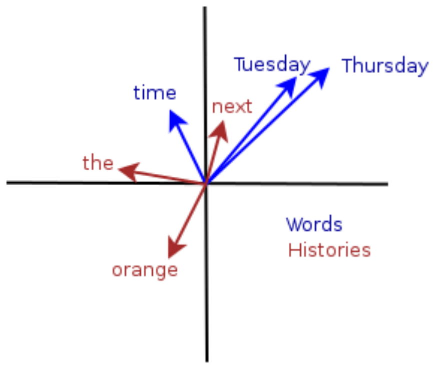


Figure 4.2: Illustration of continuous, low-dimensional representations of words (blue) and histories (red). The weighted inner products between these representations determines the probability of the word following the history. Note that similar words (e.g. days of the week) naturally cluster together; the same is true of histories.

Sparse and Low-Rank Combination

It is not plausible that all regularities in the data can be learned from a finite training set. For example, if a word is observed only a handful of times, we may simply not know enough about it to know what subspace of words it lives in. Further, some n-grams (e.g. proper nouns, idioms, etc.) do not fit into any regular pattern. A low rank matrix is not well suited to capture all of these exceptions. Thus we propose a hybrid model, where the weight matrix A is the sum of two individual components: a low rank matrix L and a sparse matrix S. The low rank component is free to model all of the regularities present in the data (a result of the inherent structure present in language). The sparse component learns the rest - the exceptions to the rule. Unlike the traditional use of sparse weights in an exponential model, our sparse matrix does not need to allocate any weights to regular, productive patterns (e.g. it does not need to model n-grams easily explained by part-of-speech constraints). Our proposed sparse plus low rank language model (SLR-LM) thus has the following form:

$$p(w|h) = \frac{\exp\left(\psi(w)^T(S+L)\phi(h)\right)}{\sum_{w'}\exp\left(\psi(w')^T(S+L)\phi(h)\right)}. \tag{4.6}$$

As a byproduct of learning, the SLR-LM separates n-grams into two qualitatively different sets: the regular low rank n-grams which are well predicted by the regular rules, and the sparse n-grams that are not. There are auxiliary benefits to this decomposition. For example, Min *et al.* [89] show that applying a sparse plus low rank decomposition directly to a word-document matrix can be used to extract document keywords; in Chapter 5 we consider similar applications of the n-grams in our sparse matrix.

4.1.2 Training Algorithm

To train the SLR-LM of Eqn. 4.6 we solve the following non-smooth convex optimization problem:

$$\min_{S,L} \gamma_0 \|L\|_* + \gamma_1 \|S\|_1 + \gamma_2 \|S + L\|_F^2 - \mathcal{L}(\mathcal{X}; S, L). \tag{4.7}$$

The entry-wise ℓ_1 norm, $\|S\|_1$, promotes sparsity in our S variables. We penalize L's nuclear norm, $\|L\|_*$, which is known to encourage low rank solutions [47, 99]. The Frobenius norm permits standard ℓ_2-norm regularization. $\mathcal{L}$ denotes the average log-likelihood, which has the familiar empirical-minus-model expectation form for its gradient as a function of A:

$$\nabla_A \mathcal{L} = E_{\hat{p}(w,h)}[\psi(w)\phi(h)^T] - E_{p_A(w,h)}[\psi(w)\phi(h)^T]. \tag{4.8}$$

In Alg. 2 we introduce an iterative algorithm for solving the above convex optimization problem. The basic structure of each iteration is to make four updates: 1) a gradient step on S, 2) an entry-wise threshold step to shrink the entries of S, 3) a gradient step on L, and 4) a singular-value threshold step on L. Both thresholding steps make use of the soft-thresholding operator:

$$\mathcal{S}_\mu(X) = \mathrm{sgn}(X) \circ \max(0, |X| - \mu) \tag{4.9}$$

where all operations are entry-wise; in particular, $\circ$ denotes entry-wise multiplication. Our algorithm is a block-coordinate variant of the accelerated proximal gradient descent algorithm introduced by Toh and Yun in [123] (see Alg. 1 in Chapter 2), modified to alternate between the ℓ_1 and $\|\cdot\|_*$ regularized terms.

4.1.3 Computational Complexity

Compared to standard exponential language model training, the SVD increases the computational cost, adding a $O(d_\psi d_\phi R_k)$ term to the per-iteration complexity, where R_k is the

Algorithm 2: TRAINSLR-LM()

```
1: $S \leftarrow L \leftarrow S' \leftarrow L' = 0; t \leftarrow t' \leftarrow 1$
2: **while** not converged **do**
3:     $mL \leftarrow L' + ((t'-1)/t)(L - L')$
4:     $mS \leftarrow S' + ((t'-1)/t)(S - S')$
5:     $mA \leftarrow mL + mS$
6:     Pick $\tau_S$
7:     $gS \leftarrow mS + (1/\tau_S)\nabla_{mA}(\mathcal{L} - \gamma_2 \|mA\|_F^2)$
8:     $pS \leftarrow \mathcal{S}_{\gamma_1/\tau_S}(gS)$
9:     $S' \leftarrow S$ and $S \leftarrow pS$
10:    $sA \leftarrow mL + pS$
11:    Pick $\tau_L$
12:    $gL \leftarrow mL + (1/\tau_L)\nabla_{sA}(\mathcal{L} - \gamma_2 \|sA\|_F^2)$
13:    $[U, \Sigma, V] \leftarrow \mathrm{SVD}(gL)$
14:    $pL = U\mathcal{S}_{\gamma_0/\tau_L}(\Sigma)V^T$
15:    $L' \leftarrow L$ and $L \leftarrow pL$
16:    $t' \leftarrow t$ and $t \leftarrow (1 + \sqrt{1 + 4t^2})/2$
17: **end while**
```

rank of L at iteration k. To speed up training, we employ many of the tricks proposed in [123], including computing partial SVDs (with PROPACK [71]) and using a continuation technique that gradually decreases the γ_0 and γ_1 weights from a large initial value to their intended target values. The τ_S and τ_L parameters in Alg. 2 are picked according to a line search analogous to Toh and Yun's. All training computation can be phrased as matrix operations, permitting us to locally parallelize computation over many cores.

4.2 English Language Modeling Experiments

4.2.1 Experimental details

All of our English experiments use telephone conversations from the Fisher corpus [41]. Each conversation in the corpus is labeled by topic; we draw data from eight of the topics (see Table 4.1) that had at least 350K word tokens. For each topic, we split the data (at the granularity of conversation) into training, development and test sets, at a 60/20/20 ratio. To avoid conflating the effects of topic and training set size, after basic text normalization we subsampled the training data (by sentence) to create three training subsets per topic, with roughly 200K, 100K and 25K word tokens, respectively. Due to limited training data, we restrict our vocabulary to the most frequency 5K word types; all out-of-vocabulary tokens are mapped to a dedicated OOV symbol.

For each topic and training set size, we train a bigram language model on the training set, use the development data to tune the regularization weights γ_0 and γ_1 (we fixed $\gamma_2 = 0$), and evaluate on the test set. Our first baseline is a "standard" ℓ_1 regularized bigram exponential (Exp) language model, trained as an SLR-LM with γ_0 chosen such that the low rank matrix is zero. Our second baseline is a modified Kneser-Ney (mKN) smoothed bigram language model.

4.2.2 Results and Discussion

The results are presented in Table 4.2. As expected, in all cases the optimal SLR-LM has a lower perplexity than the baseline ℓ_1 regularized model. Fig. 4.3, which plots the percent perplexity reduction over the Exp baseline by topic and training set size, makes it easier to see the overall trends. In particular, although gains are observed at all configurations, the biggest gains are achieved on the 100K data set size. Presumably, when there is very little training data (25K), it is difficult to learn the patterns in the data simply because too few instances are observed. In other words, we observe a rough skeleton of the true matrix, with too many holes to be accurately filled by the low rank component. On the other hand, as the amount of training data grows, there become enough examples that the patterns can be captured by the aggregation of "exceptions"; that is, the patterns in the matrix become

Topic	**# Test Tokens**	**Test OOV**	**Description**
ENG01	158K	3.5%	Professional sports
ENG02	157K	4.4%	Pets
ENG03	130K	2.9%	Life partners
ENG04	151K	3.2%	Minimum wage
ENG05	131K	3.9%	Comedy
ENG24	63K	3.6%	September 11
ENG30	76K	3.8%	Family
ENG37	81K	3.3%	Reality TV

Table 4.1: Topics used from the Fisher corpus.

	25K			**100K**			**200K**		
Topic	**SLR**	**Exp**	**mKN**	**SLR**	**Exp**	**mKN**	**SLR**	**Exp**	**mKN**
ENG01	109.4	109.9	114.6	83.0	86.5	91.6	76.7	78.8	83.4
ENG02	115.1	116.6	126.8	86.6	91.9	97.2	79.5	83.1	87.9
ENG03	116.7	119.6	125.0	88.0	92.6	96.9	80.9	83.7	88.2
ENG04	109.2	110.4	114.3	82.3	85.2	90.8	75.4	77.3	81.8
ENG05	110.2	111.3	118.5	83.4	86.4	91.9	75.4	78.4	82.9
ENG24	125.7	126.8	129.4	93.6	98.0	102.8	85.7	88.6	93.3
ENG30	114.9	116.8	123.4	86.5	90.6	96.3	79.5	83.4	86.8
ENG37	112.5	114.0	116.0	84.1	88.1	92.5	77.8	80.9	84.8

Table 4.2: Test set perplexity by topic and training set size.

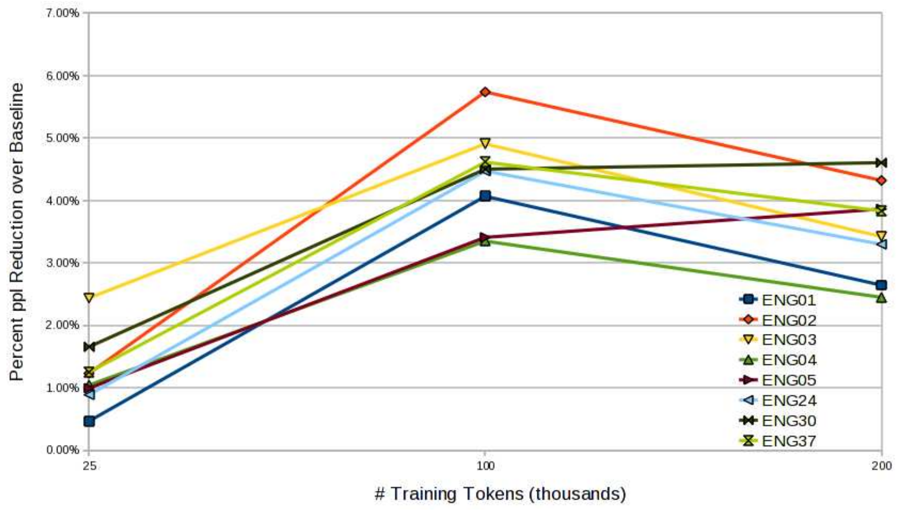


Figure 4.3: Percent perplexity reduction over Exp baseline by topic and training set size.

dense enough that there are few holes left for the low rank component to fill. Again, the SLR-LM still outperforms the standard Exp-LM in these cases, just by a smaller margin.

We looked at the high and low weight entries (n-grams) learned for the sparse and low rank components for a model trained on the topic "minimum wage." The sparse component, which models the exceptions in the data, typically learned common noun phrases, including locations ("new york", "united states") and topic-related phrases ("social security", "grocery store"). Note that nothing prevents the sparse component from having large negative weight entries to revise the probabilities of n-grams downward, although empirically this behavior is less common than positive revisions. The low rank component assigned high weights to n-grams that are syntactically plausible and semantically coherent ("you know," "I think," "I don't"), and low weights to ones that are not ("the a," "my that").

If the SLR-LM is indeed learning a low-dimensional continuous representation, words (or histories) that function similarly should be mapped close to each other in the continuous representation. Using a model trained with 200K tokens on the topic of "pets," we list in Table 4.3 several words and histories with their nearest neighbors in the low dimension space. Note that sometimes a word's neighbors are similar in either position (e.g. "would"), but they need not be (e.g. "but"). As expected, natural clusters form, e.g. numbers.

History Word h	**Nearest Neighbors**
would	could, didn't, can't, don't, should
but	well, mean, because, guess, think
of	from, for, at, in, make
Prediction Word x	**Nearest Neighbors**
would	can, did, don't, didn't, should, could
but	so, because, now, for, as
know	keep, think, want, get, talk
one	two, four, three, ten, five
dog	cat, thing, ones, animal, baby

Table 4.3: Words, histories, and their nearest neighbors in the continuous-space embeddings induced by the low rank component from the 200K ENG02 set.

Although our approach to finding low dimensional representations bears a superficial similarity to Latent Semantic Analysis (LSA) [42], they are learned in very different ways and optimize different objectives. LSA compiles a word-document co-occurrence matrix and takes a single truncated singular value decomposition, which minimizes the Frobenius norm between the original and low rank matrices, to induce semantic similarity between words based on their co-occurrence within documents. In contrast, training our model iteratively learns low dimensional representations of words and histories jointly in order to maximize the log-likelihood of a parametric distribution whose probabilities are governed by weighted inner products between the low dimensional representations. In our model, the singular value decomposition is computed on the low rank weight matrix, and not on any empirical counts from the data directly.

4.3 Low Resource Language Modeling Experiments

As the English language modeling experiment results in Table 4.2 indicate, the SLR-LM can aid performance when training data is limited (which is when exploiting the similarities between words and histories is most beneficial). For English, many common scenarios do not involve limited training data; in fact, a fair amount of English language modeling research involves scaling models to massive datasets. However, limited training data is the norm for many so called "low resource languages," which lack the corpora and tools that are available for languages like English, Mandarin and Modern Standard Arabic. In this section we conduct language modeling experiments in low resource conditions.

We employ corpora from a diverse set of five languages (Cantonese, Pashto, Turkish, Tagalog and Vietnamese) to evaluate the SLR-LM's performance in the limited data condition. Our corpora consist of transcribed telephone conversations and are distributed through the IARPA Babel program[3] Specifically, we use the "Limited Language Pack" version of the data for each language, where only roughly 10 hours of speech is available for training. After text normalization to lower-case the data and remove word fragments, unintelligible speech and most non-speech tokens, our data sets statistics are summarized in Table 4.4. Note the large variation in vocabulary size and OOV rate.

For each language we train a bigram SLR-LM. Unlike previous experiments, OOV words are not mapped to an OOV token: our models never predict OOVs and they break history context (OOVs at test time are not included in the perplexity computation). At a very coarse level we tune the regularization weights (γ_0, γ_1 and γ_2) on a small development set, and evaluate on the test set. As a baseline, we also train modified Kneser-Ney bigram models on the language models, treating OOVs in the same fashion.

The perplexity results of our model are listed in Table 4.5. On all languages the SLR-LM beats the baseline model, with an average of a 7.1% reduction in perplexity. This reduction is consistent with the reduction observed in English, and supports the idea that the SLR-LM is an appropriate model to use when training data is limited.

4.4 Comparison with other Continuous Space Models

As discussed in Chapter 2, there are other continuous-space language models. Though we do not have direct comparisons to these models on our data, we present in this section some

[3]We use the Cantonese language collection release babel101b-v0.4c_sub-train1, the Pashto language collection release babel104b-v0.4bY, the Turkish language collection babel105b-v0.4, the Tagalog language collection release babel106b-v0.2g-sub-train, and the Vietnamese language collection release babel107b-v0.7.

Language	# Training Tokens	# Test Tokens	Vocab Size	OOV Rate
Cantonese	98.9k	17.0k	5.0k	7.4%
Pashto	111.0k	36.4k	6.2k	5.2%
Turkish	70.8k	24.1k	10.1k	15.4%
Tagalog	68.3k	23.4k	5.5k	9.1%
Vietnamese	115.5k	41.2k	2.9k	1.4%

Table 4.4: Statistics of the low resource language data used in our language modeling work, which come from the "Limited Language Pack" portions of the data collection releases for the IARPA Babel program.

Language	mKN Perplexity	SLR-LM Perplexity
Cantonese	112.7	103.3
Pashto	159.0	145.9
Turkish	246.5	234.6
Tagalog	123.3	113.0
Vietnamese	176.7	166.2

Table 4.5: Language modeling performance across languages, comparing a modified Kneser-Ney (mKN) baseline to the SLR-LM.

		Ppl Reduction	
Model	n	Alone	Interpolated
Neural Net (NN)	5	0.7%	17.4%
Log-Bilinear	5	-2.3%	18.4%
Recurrent NN	-	11.7%	25.1%
RNN-LDA	-	19.5%	30.4%

Table 4.6: Perplexity reductions reported in [85] with the Penn Treebank corpus (930K training tokens).

		Ppl Reduction	
Model	n	Alone	Interpolated
NN	4	9.2%	-
Deep NN	4	10.1%	19.3%

Table 4.7: Perplexity reductions reported in [6] with the Penn Treebank corpus (930K training tokens).

recent results from the literature to give context for the kinds of gain researchers achieve over baselines.

In Table 4.6, we summarize results included in [85]. The "Alone" column lists the relative reduction in perplexity over a baseline modified-Kneser-Ney 5-gram using the listed model only, while the interpolated column shows the improvement when the individual model is also interpolated with the baseline. Alone, the neural network language model gives very little gain, and the log-bilinear model [90] actually hurts performance. It is only with interpolation do they get large gains. In contrast, the reductions we reported in previous sections, between 5.6% to 9.5%, are obtained using our model alone with no interpolation. Zweig and Mikolov report two recurrent neural network (RNN) results: the first a "standard" RNN and the latter one that also uses topic information as learned by latent Dirichlet allocation [14] as an input feature. Both give strong improvements, particularly with interpolation. These large improvements are typical of recurrent neural network models, though as mentioned in Chapter 2, the arbitrary history length precludes their practical use in speech recognition decoding and lattice rescoring. Table 4.7 reports a few additional data points from [6] on a larger training set, showing improvements around 10% alone. Again, the improvements reported in Tables 4.6 and 4.7 are not directly comparable to ours, since they use different datasets (with different sizes, languages and genres), but do give a sense of the kinds of improvements being obtained by state-of-the-art continuous space models.

In Table 4.8, we summarize the relative advantages and disadvantages of the SLR-LM relative to other classes of language model. The "Pros" refer to places where the SLR-LM is stronger, and "Cons" where it is at a disadvantage. That the SLR-LM learns sparse exceptions, which will be shown to be meaningful for other language processing tasks, is an advantage over all of the comparison language models listed. Relative to n-gram language models in particular, the SLR-LM is better able to smooth, in part due to its continuous space nature and in part to its ability to incorporate features. This comes at the cost of training complexity and ease of integrating into speech recognition decoding, both of which favor the n-gram model. Compared to maximum entropy and Model M language models, the SLR-LM has the advantage of being a continuous space model, but is slower to train and requires more memory. In favor of the SLR-LM relative to neural network language models (include deep and recurrent ones) is its ability to use features of the prediction word, its convex training objective and that it learns the dimensionality of the continuous representation automatically. Unlike the deep or recurrent neural networks, the SLR-LM is unable to learn deep representations of histories. Unlike recurrent neural networks, the SLR-LM can only exploit arbitrarily long histories through features. Finally, compared to the log-bilinear model, the SLR-LM is convex and learns the dimensionality of the continuous representation, but since it does not factor the weights, it is less scalable. There are some attributes that could be interpreted as either an advantage or disadvantage. For example, the SLR-LM has a simpler form compared to neural network language models, which makes training easier but limits the capacity of the continuous space mapping. Likewise, the fact that recurrent neural network language models have arbitrarily long histories is an advantage

SLR-LM Compared to Standard n-gram LMs

Pros	Cons
Able to exploit similarities (better smoothing)	Far less scalable
Supports features (word and history)	Harder to integrate into decoding
Able to identify exceptional n-grams	

SLR-LM Compared to Maximum Entropy and Model M LMs

Pros	Cons
Able to exploit similarities (better smoothing)	Less scalable
Able to identify exceptional n-grams	

SLR-LM Compared to Neural Network LMs

Pros	Cons
Convex (easier training)	Cannot learn deep representations
Able to incorporate features of prediction word	Harder to exploit arbitrarily long history
Dimensionality of continuous representation is learned	
Able to identify exceptional n-grams	

SLR-LM Compared to the Log-Bilinear LM

Pros	Cons
Convex (easier training)	Less scalable
Dimensionality of continuous representation is learned	
Able to identify exceptional n-grams	

Table 4.8: Summary of key differences between the SLR-LM and other language models. Pros refer to advantages of the SLR-LM; cons refer to limitations of the SLR-LM.

in terms of modeling power, but limits the scenarios in which they can be used.

4.5 Turkish Morphological Language Modeling Experiments

Highly agglutinating languages are challenging to model due to the substantial vocabulary sizes. One common strategy for grappling with this challenge is to incorporate morphological information, allowing information to be pooled over related sets of words (e.g. those with a shared inflectional affix). One nice property that the SLR-LM shares with the ME-LM is the ability to incorporate arbitrary features of words and of histories [100]. In this section we explore the use of morphology as features in a SLR-LM on conversational Turkish data, which exhibits extensive agglutination. It is important to note that the units of our language model are *words*, not morphemes; we use morphological information only as a feature.

There are several ways in which morphology can be used as features. If the morphology is concatenative, one might decompose a word into its constituent morphemes and treat the presence of a morpheme as a feature for that word; for example, by representing a word as an indicator vector of its morphological constituents. In highly agglutinating languages, it may be reasonable to treat sequences of affixes as a single *affix bundle*, in which case there would be features indicating the stem and any affix bundles (prefix or suffix). Non-concatenative morphological phenomena can be handled as well; for example, in language with reduplication a single binary reduplication feature can be added to the morphological feature vector to indicate whether the word does or does not contain reduplication. In cases of templatic morphology, as in Arabic, one might indicate the stem and the template. Beyond the issue of the way in which a word can be decomposed into morphological elements, there is the issue of how and when to include it as a feature. One can use only features of the history or features of the word, or both. Empirically in pilot studies, we find that using features of the word can lead to overly smoothed probability distributions. Features of the history typically decompose into features over individual history words. The features of the history words can be stacked (so that the overall history vector is the concatenation of per-history-word feature vectors), which preserves sequencing information, or they can be merged in a “bag-of-morphs” style, where one only indicates whether a morpheme appeared

babandan	Original Form
babLndLn	After vowel harmony normalization
babL +n +dL +n	After morphological decomposition

Figure 4.4: Example Morfessor morphological decomposition of Turkish word "babandan" (father). This decomposition yields one stem feature ("babL") and two suffix features ("+dL" and "+n") for the word.

or not *anywhere* in the history window.[4] The effectiveness of the two styles will depend on the language: the freer the word order the more appropriate the bag-of-morphs modeling assumption is. The bag-of-morphs style has a computational advantage, since it leads to a lower history feature dimension.

4.5.1 Experimental details

Data

Our Turkish data comes from the IARPA Babel project (Turkish data collection release babel105b-v0.4) and consists of telephone conversation transcripts. For language modeling, we use the provided "Limited Language Pack" (LLP) training set which, after removing non-speech tokens and lower-casing, has a 10k word vocabulary despite only containing 70k training tokens. We tune our models using a 23k word development set that is a subset of the official 70k development set, and evaluate on a different 24k word subset (the subsets were automatically selected to balance dialect and gender characteristics).

Morphological Decomposition

We use an unsupervised morphological decomposition learned on the "Full Language Pack" (IARPA Babel Turkish data collection release babel105b-v0.4) which consists of 38k word types and 554k word tokens.[5] It uses the Morfessor tool [36] to learn the decomposition,

[4]We thank Janet Pierrehumbert for the suggesting features that ignore word order, motivated by Turkish.

[5]In addition to our appreciation for providing this decomposition, we would also like to thank Peter Baumann and Janet Pierrehumbert for their advice on choices of features and insights in the interpretation

after the vocabulary was first pre-processed to normalize orthography for vowel harmony (another notable feature of Turkish) [7]. Fig. 4.4 illustrates the way in which a word might be decomposed into morphological constituents. Each distinct morpheme will serve as a feature, as described next.

Features

In these experiments, our representation of the history, $\phi(h) = \left[\hat{\phi}(w_{-(n-1)})^T, \ldots, \hat{\phi}(w_{-1})^T\right]^T$, is the concatenation of vector representations of each word in the history. The vector representation of the kth most recent history word, $\hat{\phi}(w_{-k})$, is $(V + M)$-dimensional, where V is the number of words in the vocabulary and M is the number of distinct sub-word units (e.g. morphemes) observed in the decomposition of the vocabulary. Then $\hat{\phi}(w_{-k})$ is a sparse binary vector with a 1 indicating the word identity and 1s indicating each of its constituent sub-word units. As our experiments will show, feature pruning is an useful step in order to obtain good performance. We rely on three criteria for pruning:

1. First, a morphological feature must appear in a sufficient number of word types. Without any cutoff, it is possible that a morphological feature only appears in a single word type, in which case it provides no additional information over the word identity itself.

2. Second, a morphological feature must appear in a sufficient number of word training tokens. This simply ensures that the weights for the feature can be estimated effectively.

3. Finally, we also consider excluding certain classes of morphemes; e.g. stems. For example, with a bigram model, the local sequencing is largely influenced by part of speech, which is better captured with affix morphemes or affix bundles. In contrast, stems are poor for local sequencing information, because they can be present with different parts of speech, reducing their utility as a predictor of an upcoming word.

of the results.

Morpheme Units	Type Cutoff	Token Cutoff	Perplexity
None	-	-	234.7
2g stem, 2g affix	None	None	237.2
2g stem, 2g affix	5	20	235.3
2g affix	None	None	232.3
2g affix	5	20	**231.6**
10g B-o-S, 2g affix	None	None	**231.6**
10g B-o-S, 2g affix	5	20	233.1

Table 4.9: Language modeling results with different morphological feature sets. ng denotes n-gram and B-o-S denotes "bag of stems." A bigram modified-Kneser Ney model on words gives a perplexity of 246.5, while a bigram Factored Language Model with affix features gives a perplexity of 241.9.

The thresholds for criteria 1 and 2 are loosely tuned on our development set.

4.5.2 Results and Analysis

Using the LLP training data, we estimate train several variants of morphological-feature SLR-LMs. The results are listed in Table 4.9. First, the baseline SLR-LM that uses no morphological features achieves a perplexity of 234.7. The second block of the table includes results with the unsupervised Morfessor decomposition features. Adding bigram stem and affix features without pruning yields a perplexity of 237.2 (though this is improved to 235.3 when stem and affixes are pruned by type and token frequency). Excluding stem features, which risk conflating noun, verb and other part-of-speech contexts, improves performance to 232.3. Finally, with a pruned affix feature set yields a perplexity of 231.6. A 10-gram bag-of-stem features with bigram affix features neither helps nor hurts performance, despite needing no pruning. When the features are pruned, performance slightly degrades. These results outperform our two baselines: a bigram modified-Kneser Ney model on words, which gives a perplexity of 246.5, and a bigram Factored Language Model with affix features, which gives a perplexity of 241.9.

To gain intuition into how morphological features are being used in the model, we plotted the low rank and sparse weights for morpheme and word features (i.e. the ℓ_2 norms of the rows of L and S) for the "2g affix" model with token and type cutoffs of 20 and 5, respectively. As shown in Fig. 4.5, morphological features are more naturally modeled by the low rank matrix (in comparison to word features). This distinction between morpheme and word features suggests two possible explanations: 1) the way that morpheme and words function in the history may be qualitatively too different to effectively map down into the same low-dimensional space, forcing the low rank and sparse matrices to model different kinds of features, and 2) the morphological features yield more regular sequential patterns, and are thus more naturally suited to the low rank matrix.

Ours is not the first model to incorporate morphological features. One of our baseline models, the Factored Language Model (FLM) [12], was discussed in Chapter 2. It maps each word to a set of factors; for example, mapping an Arabic word to a stem, root and pattern. Our approach shares with the FLM the idea that expressing a word as a set of sub-parts can lead to more reliable estimation for morphologically rich languages. There are a number of key differences, though: the SLR-LM is an exponential model while the FLM is an n-gram model, the FLM assumes a fixed number of "factors" while in the SLR-LM a word can map to a variable number of features (as long as the feature vector dimension is fixed), and the SLR-LM provides a mechanism for learning similarities and exceptions within the morphological feature space, which is not supported by the FLM. The Factored Neural Language Model [2] uses morphology more closely to the way we use it; namely, it treats morphology as features. The modeling assumptions are fairly different, though, due to the underlying differences between neural networks and exponential models, and while FNLM can learn low-dimensional representations of morphological features (in the history, at least), it has no mechanism for learning exceptions. Others have introduced sophisticated and effective language models in the exponential framework that use morphological information [108, 103], but like the FNLM, they do not learn low dimensional representations nor can they find exceptions.

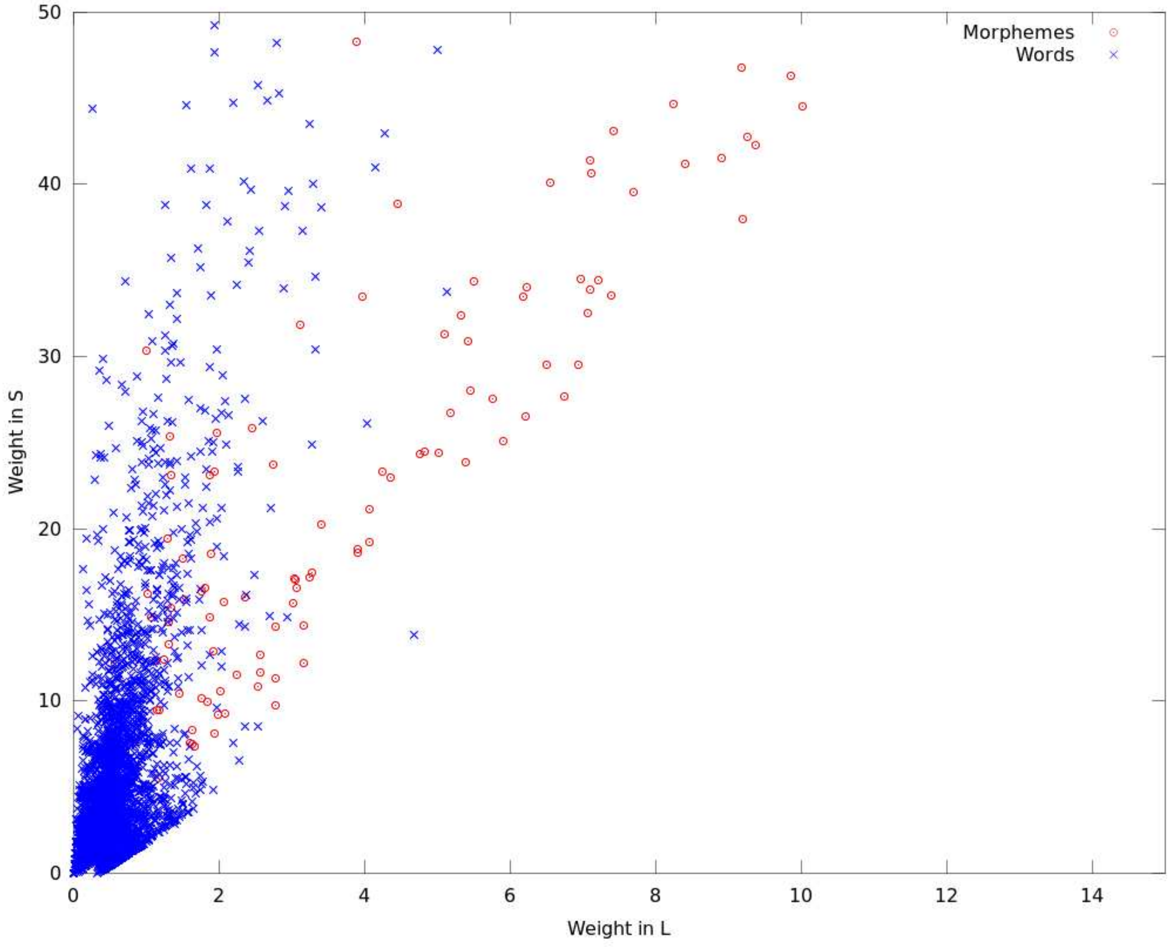


Figure 4.5: For each history feature in a trained morphological SLR-LM, the ℓ_2 norm of the weights are plotted (morphological features in red circles; word features in blue x's).

4.6 Word and Multiword Learning Experiments

In the writing systems of most languages, words are delimited with white space, but this is not universal. Mandarin and Cantonese, for example, are often written without any whitespace. For consistency, Vietnamese is often written at the syllable level. In both cases, the easiest unit to work with is the syllable, since this is the only deterministic segmentation available that does not rely on external linguistic knowledge. This is problematic for speech recognition for two reasons: first, syllable-based language models tend to be weak, since for any given n the n-gram context covers less of the full history, and second, shorter units have fewer distinguishing components and are thus more prone to accidental insertion, deletion or substitution. It is desirable, therefore, to take a syllable-level representation of language and automatically convert it into one that includes multi-syllable units, which typically would be word or multiword units. It is even better for the low-resource case if the technique for identifying words and multiwords is entirely automatic and makes no use of language knowledge. This is the problem we tackle in this section.

One very important characteristic of the SLR-LM is that its weight structure reveals something about language; in particular, the sparse plus low rank structure decomposes language into exceptional n-grams and productive n-grams. In this section, we see how the sparse elements in a language model over sub-word units (i.e. syllables) can automatically identify words and multiwords in the language. We compare what it learns against and combine with several other state-of-the-art techniques, and evaluate both in terms of language modeling performance and in the rate at which it identifies actual words.

4.6.1 Experimental details

As mentioned above, we will consider two syllable segmented languages: Cantonese and Vietnamese. Our data come from the IARPA Babel program (Cantonese data collection release babel101b-v0.4c_sub-train1 and Vietnamese data collection release babel107b-v0.7). We use the “Full Language Pack” training sets with roughly 100 hours of training data per language. The Vietnamese data comes syllable segmented, while for Cantonese the data comes word segmented. To simulate more typical conditions, we converted the Cantonese text into a character-segmented form (leaving foreign words with roman character sequences intact). The data set and vocabulary sizes are summarized in Table 4.10.

Using the training data, we generate several ranked lists of candidate multiwords according to the following methods.

- SLR-LM (`slrlm`). To obtain these, we first train a bigram SLR-LM on the syllable- or character-level data. After the model has been tuned, we take the positive elements of the sparse weight matrix to be our candidate (bigram) multiwords. The list is sorted by the magnitude of the entries, from largest to smallest.

- Bigram product (`bp`). Drawing from the literature on multiword learning [102], we compute the bigram product $p(w_1, w_2)/\sqrt{(p(w_1)p(w_2))}$ for all bigrams $w_1 w_2$ in the training data, and sort the list from largest to smallest. (Bigram product was shown to work better than mutual information in [102].) With some manipulation, this can be shown to give an equivalent ranking as taking the product of the probability of the first word given the second and the probability of the second word given the first. In either form, it clearly favors bigrams whose relative frequency of occurring together is high.

- Adaptor Grammar (`ag`).[6] Adaptor Grammars [62] are a framework for defining a variety of non-parametric hierarchical Bayesian models that delivers state-of-the-art performance on unsupervised word segmentation of phonemically transcribed child directed speech [63]. Here we investigated two different "collocation" adaptor grammars for resegmenting the training data. Both these models capture inter-word dependencies by learning larger-than-word units (see [65] for details). The resulting segmentation is used to identify potential multiwords to be added to the lexicon. Following the methodology outlined in [65], we used minimum Bayes risk decoding and put fixed priors on all hyper-parameters instead of manually tuning model parameters.

[6]We thank Mark Johnson and Benjamin Börschinger both for providing our Adaptor Grammar multiword candidate lists and for providing this paragraph describing Adaptor Grammars.

Language	Units	Training Tokens	Dev Tokens	Test Tokens	Vocab Size
Vietnamese	Syllables	890k	40.7k	38.7k	5.2k
Cantonese	Characters	970k	21.1k	78.1k	3.5k

Table 4.10: Description of data for the word and multiword learning experiments. For Cantonese, the character-level segmentation is obtain by splitting an original word segmentation to simulate unsegmented conditions.

- Frequency (`freq`). This simple baseline sorts all bigrams in the training data by their frequency of occurrence.

- Pronunciation variability (`pron`).[7] This method finds adjacent token sequences that most often exhibit non-standard pronunciations, where non-standard pronunciations are found by first performing a forced alignment of the training text, and then marking as non-standard any candidate multiword that contains at least one phoneme with pronunciation variation (defined to be phones whose average per-frame acoustic score is at least two standard deviations under the model mean for that triphone).

- Voting. We consider a simple voting strategy, where several multiword lists are taken as input. Each list is truncated to the top K candidate multiwords, and then each of the lists "vote" for their K multiwords. The combination sorts multiwords by vote count, so any candidate multiword that appeared in all lists would appear before any candidate multiword that appeared in all but one list, and so forth. Multiwords with the same vote count are sorted by their bigram product score.

- Zipper. An even simpler combination method is also considered. This combination approach walks round-robin through the input lists, first taking the top entries in each, then the second entries in each, and so forth (candidate multiwords are only added the first time they are encountered).

[7]We thank Rohit Prabhavalkar, Yanzhang He and Eric Fosler-Lussier at Ohio State University for conducting the pronunciation variability analyses and for providing the corresponding lists.

4.6.2 Results and Discussion

We first evaluate how well each method does at identifying words in the language. Specifically, for each list, we take the top N entries and see which percentage of the candidate multiwords (recall that these are actually multisyllables or multicharacters) appear in an external dictionary,[8] and vary N. Note that precision with an external dictionary is an imperfect criterion, since there is no guarantee that the dictionaries have perfect coverage, especially of valid multiwords. In a limited analysis, we took the top 25 hypothesized words for the `ag`, `slrlm` and `bp` methods that *were not* in the dictionary and asked native speakers of Cantonese and Vietnamese to evaluate them.[9] The native Vietnamese speaker indicated that 84% of the hypothesized words were actually valid words or multiwords. In Cantonese, 58% were deemed valid words (often nouns shared with Mandarin), 15% deemed valid multiwords (typically Cantonese spoken expressions) and 27% neither valid words nor multiwords. Because of the noisy nature of this statistic, we consider only large differences meaningful.

The results are shown in Figs. 4.6 and 4.7. Our analysis of the dictionary does not suggest that fixing the dictionary would bias in favor of one method or the other, but does suggest that the performance is stronger in general than these figures indicate. For Cantonese, the voting combination of `bp`, `slrlm` and `ag` is the clear winner for all list sizes. Selecting multiwords by pronunciation variability performs poorly; it may be that the variation is often attributable to dialect instead of within-word reduction. The frequency criterion, as expected, is quite poor at identifying words. The overall trends are quite similar for Vietnamese, with the exception that voting does not help and the `ag` method is somewhat weaker.

Next, we evaluate how word and multiword learning can be used to improve language modeling. By selectively joining syllables or characters into words, both the vocabulary size and the average effective history context increases. The process is simple: 1) given a multiword list we resegment the training and test data, joining pairs of syllables or characters

[8]Cantonese dictionary: http://kaifangcidian.com/xiazai/ (33.3k words).
Vietnamese dictionary: http://vlsp.vietlp.org/ (31.1k words).

[9]We thank Yanzhang He and Van Hai Do for performing these analyses.

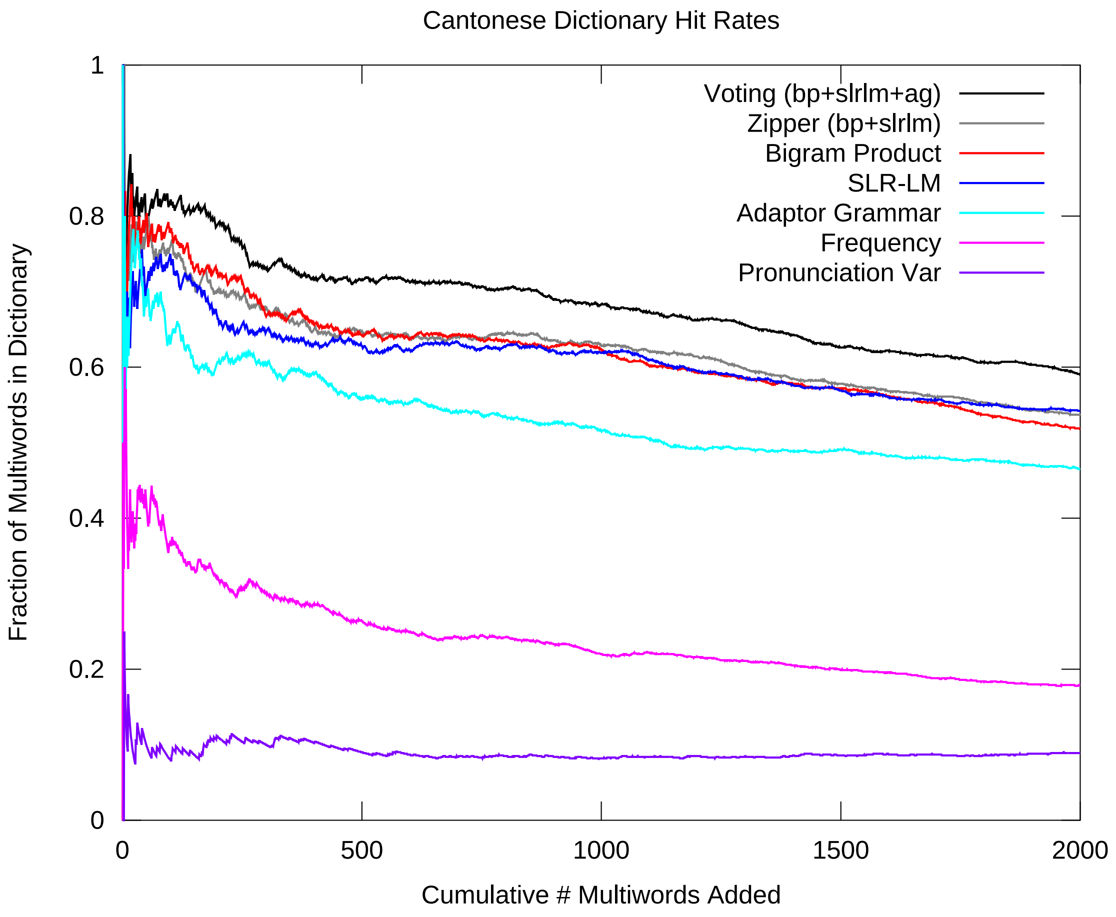


Figure 4.6: The proportion of Cantonese “multiwords” (words) found in an external dictionary as the number of multiwords grows, for different multiword lists. Methods compared include SLR-LM, Bigram Product, Adaptor Grammar, Frequency, Pronunciation variability and two combination methods: a voting combination of bp, slrlm and ag (Voting) and a zipper combination of bp and slrlm (Zipper).

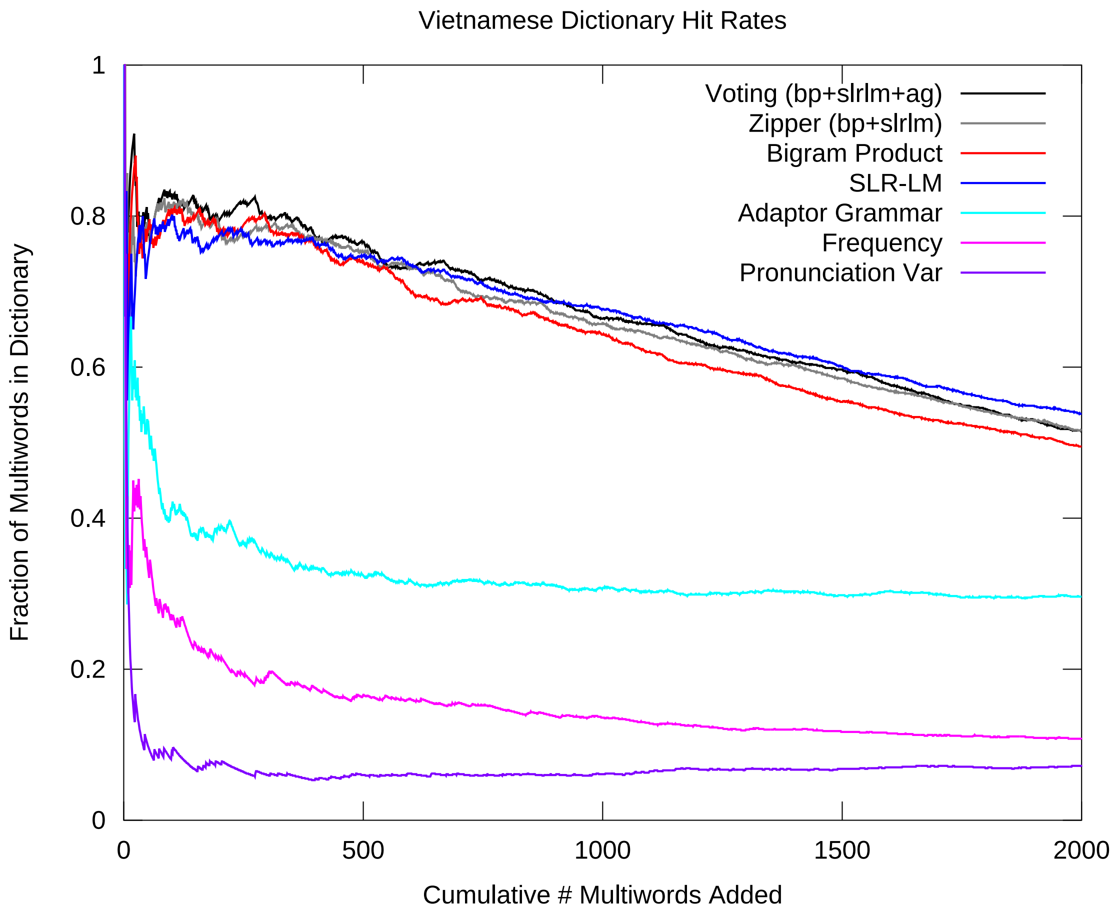


Figure 4.7: The proportion of Vietnamese "multiwords" (words) found in an external dictionary as the number of multiwords grows, for different multiword lists. Methods compared include SLR-LM, Bigram Product, Adaptor Grammar, Frequency, Pronunciation variability and two combination methods: a voting combination of bp, slrlm and ag (Voting) and a zipper combination of bp and slrlm (Zipper).

Multiword Method	Cantonese Test ppl	Vietnamese Test ppl
No Multiwords	74.2	138.5
`slrlm`	73.1	138.0
`bp`	72.3	137.9
`ag`	73.1	141.2
`freq`	80.1	148.2
`pron`	75.3	139.7
`v-bp+slrlm+ag`	72.4	137.9
`z-bp+slrlm`	**72.0**	**137.8**

Table 4.11: Perplexity after resegmenting the training and test data using the specified set of multiwords.

that are elements of the multiword list, 2) a language model is trained on the resegmented training data, and 3) the perplexity is computed on the resegmented test data. In order to compare perplexities with the different vocabularies, we normalize our perplexity here not by the number of "words" in the text, but by the number of syllables or characters in the text, making comparisons across conditions meaningful.

In order to resegment the text, a fixed list size needs to be established. In Figs. 4.8 and 4.9 we plot the perplexities for a subset of the methods over list size. To establish a fixed list size, we choose the list size that minimizes the median perplexity over the five methods, which results in a list size of 1400 for Cantonese and 1200 for Vietnamese. With the exception of the `ag` method for Vietnamese, there is a range of sizes that give similar performance as the vocabulary sizes chosen; perplexity is relatively flat in those regions. Given the fixed list sizes, we summarize the perplexity for several lists in Table 4.11. Small reductions in perplexity are obtained by `slrlm`, `bp`, `ag` and the combination methods. The best result is obtained with the zipper combination, but the differences relative to individual methods is probably not significant. Which method is best needs to be answered in the context of a downstream application (e.g. speech recognition), which serves as future work.

Finally, we conclude with an analysis of how similar the different lists are to each other.

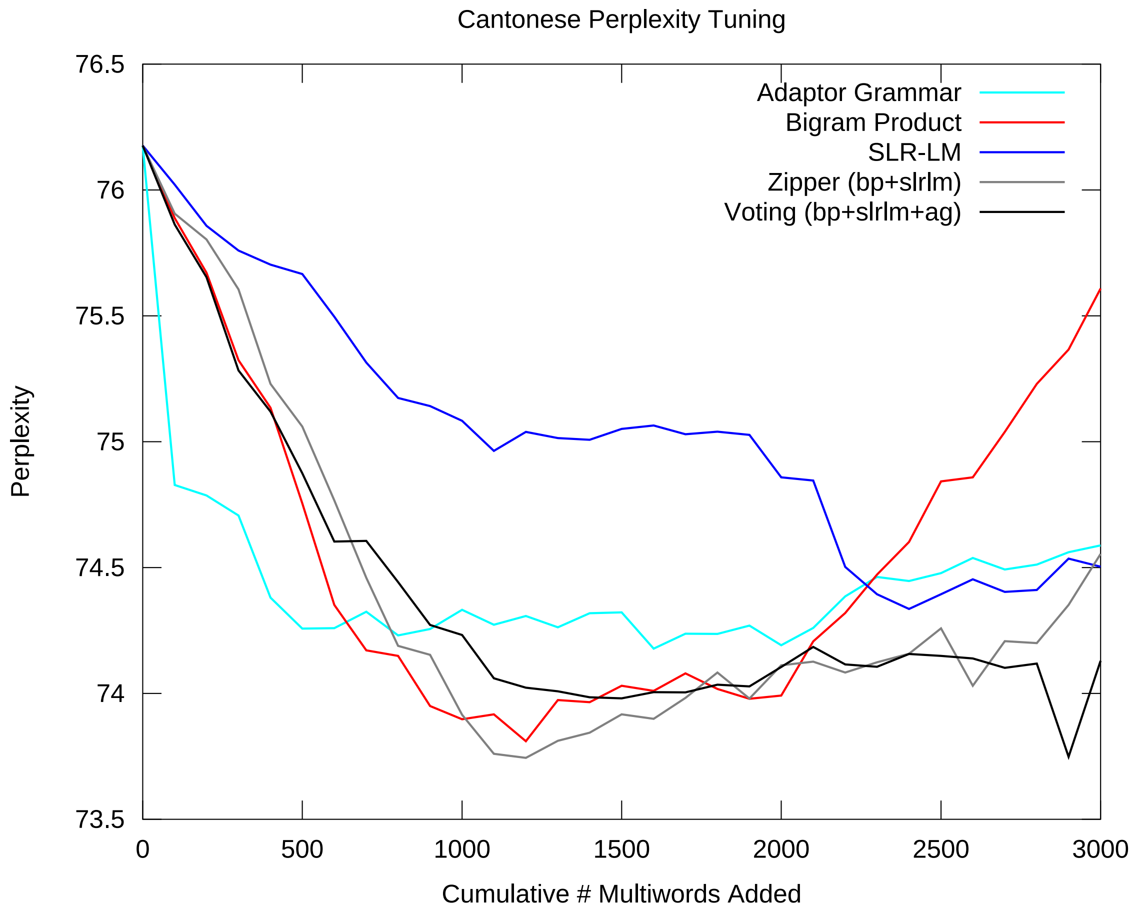


Figure 4.8: The dev set perplexity of Cantonese language models using resegmented data. Five multiword lists are considered: Adaptor Grammar, Bigram Product, SLR-LM and two combination methods: a voting combination of bp, slrlm and ag (Voting) and a zipper combination of bp and slrlm (Zipper).

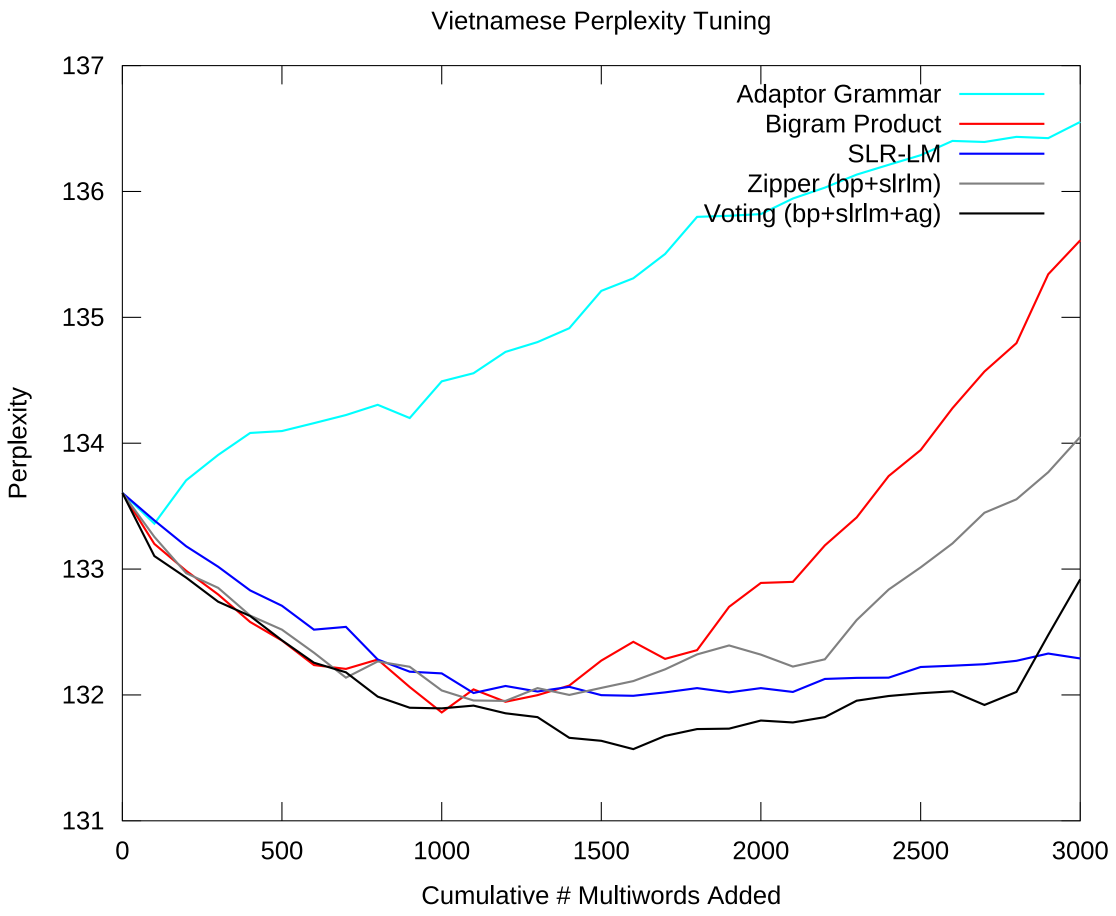


Figure 4.9: The dev set perplexity of Vietnamese language models using resegmented data. Five multiword lists are considered: Adaptor Grammar, Bigram Product, SLR-LM and two combination methods: a voting combination of bp, slrlm and ag (Voting) and a zipper combination of bp and slrlm (Zipper).

Since each list is ordered, we can convert it into a ranked list and use Spearman's rank correlation coefficient ρ to compute pairwise correlations between each lists, where

$$\rho = \frac{\sum_{i=1}^{N}(x_i - m_x)(y_i - m_y)}{\sqrt{\sum_{i=1}^{N}(x_i - m_x)^2 \sum_{i=1}^{N}(y_i - m_y)^2}}. \tag{4.10}$$

Here x_i is the rank of ith candidate multiword in the first list and y_i is the rank of the ith candidate multiword in the second list. All candidate multiwords whose scores are originally tied (e.g. all multiwords in the `freq` list that appear the same number of times) are assigned the same rank, which is the average of their original rank values. m_x and m_y are the average values of the first and second lists, respectively. N is the number of candidate multiwords appearing in either list. The set of candidate multiwords in the first list but not the second list are appended to the end of the second list, sharing the same "tied for last" rank; words in second list but not the first list are handled analogously. The pairwise correlations between lists (after truncated to the fixed list lengths determined earlier) are shown in Table 4.12. The only positive correlation is between `slrlm` and `bp`. The correlations are visualized using multi-dimensional scaling in Fig. 4.10, which makes it clear that in these fixed length lists there are three sets: 1) `bp` and `slrlm`, 2) `freq` and `ag` and 3) `pron`. These sets are roughly compatible with the behaviors of the methods in the dictionary hit rate experiments. It is noteworthy how similar the trends are for Cantonese and Vietnamese.

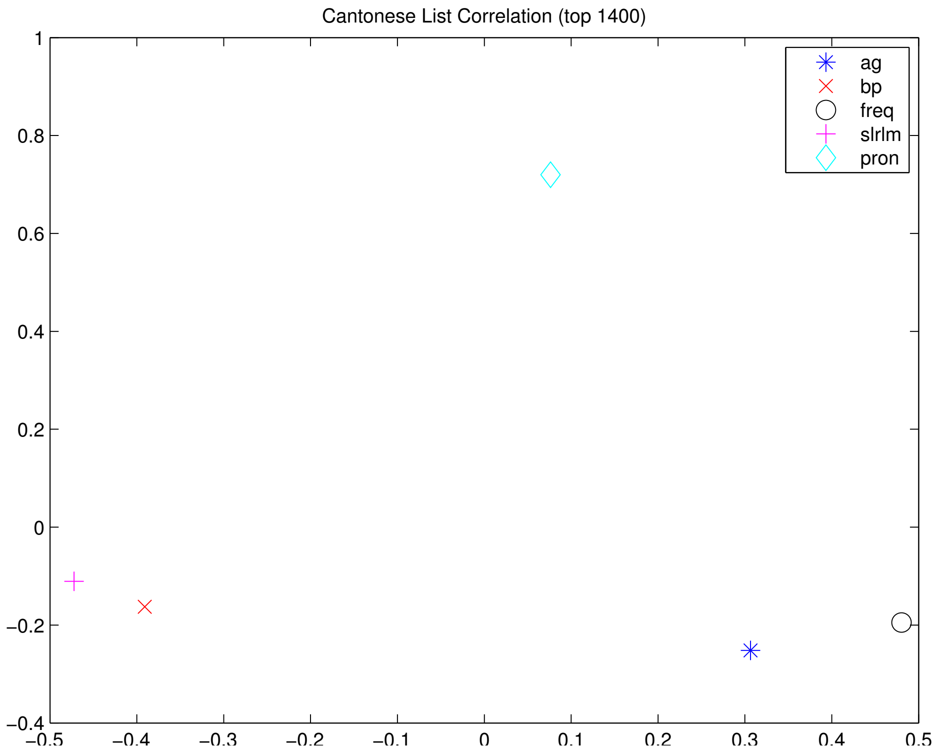
4.7 Conclusions

In this chapter we introduce a new sparse plus low rank language model that generalizes existing, popular ℓ_1-regularized exponential models, and an efficient algorithm to train it. The SLR-LM automatically performs natural and flexible "soft-tying" of parameters between similar words (and histories) that improves generalization, and can be viewed as a continuous space language model trained in the exponential framework. In English language modeling experiments on conversational speech, with varying topic and training set sizes, we observe consistent 2-5% reductions in perplexity over a maximum entropy baseline, and 5-9% reductions over a modified Kneser-Ney baseline. Motivated by previous results [58, 101] and those summarized in Tables 4.6 and 4.7, we expect that interpolating the SLR-LM with

(a)	ag	bp	freq	slrlm	pron
ag	1.000	-0.267	-0.120	-0.502	-0.856
bp	-0.267	1.000	-0.550	0.669	-0.798
freq	-0.120	-0.550	1.000	-0.725	-0.857
slrlm	-0.502	0.669	-0.725	1.000	-0.785
pron	-0.856	-0.798	-0.857	-0.785	1.000

(b)	ag	bp	freq	slrlm	pron
ag	1.000	-0.371	-0.029	-0.580	-0.855
bp	-0.371	1.000	-0.617	0.741	-0.807
freq	-0.029	-0.617	1.000	-0.766	-0.857
slrlm	-0.580	0.741	-0.766	1.000	-0.801
pron	-0.855	-0.807	-0.857	-0.801	1.000

Table 4.12: Pairwise Spearman's correlation for Cantonese with the top 1400 entries in each list (a) and Vietnamese with the top 1200 entries in each list (b).



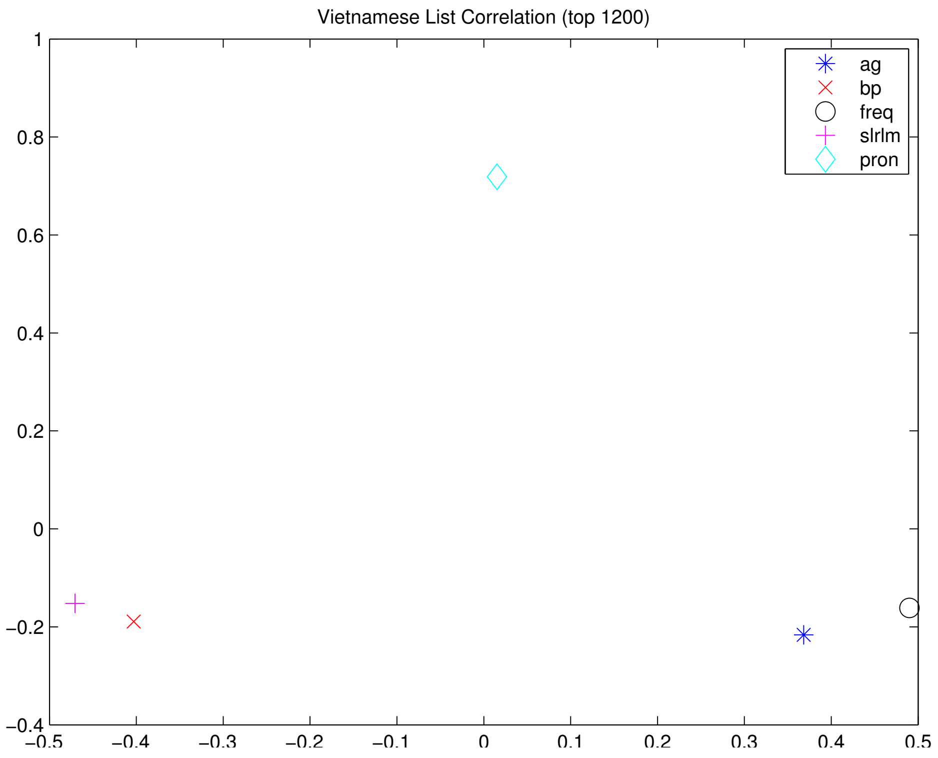


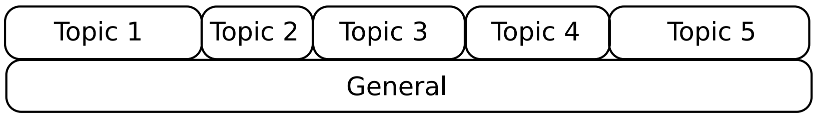
Figure 4.10: A visualization of the pairwise Spearman's correlation for Cantonese with the top 1400 entries in each list (top) and Vietnamese with the top 1200 entries in each list (bottom).

a standard smoothed n-gram model would yield further improvements. To facilitate interpretation of the model, we first focused on basic n-gram features and an English language model task, but followed up with experiments on several low resource languages: Cantonese, Pashto, Turkish, Tagalog and Vietnamese. We find an average reduction in perplexity over the modified Kneser-Ney baseline of 7.1%, consistent with the English results. Though the training set sizes limited our models to bigrams, feature functions allow one to trivially model higher order n-grams (e.g. by letting $\phi(h)$ map to a one-hot encoding of (n-1)-grams). Since the SLR-LM supports arbitrary feature functions on words and on histories, we conducted a set of experiments using morphological features for morphologically-rich Turkish, finding that the use of morpheme features can bring an improvement over word features alone. The Turkish experiments also point to an intuitive result that morphological sequential behavior is more reliably estimated from a small training set than word sequential behavior. Finally, we investigate the ability of the SLR-LM to automatically learn words and multiwords from character-segmented Cantonese and syllable-segmented Vietnamese, finding that SLR-LM (possibly in combination with standard, existing methods) does a better job than our baseline methods at finding dictionary words, and when used to re-segment the training data is able to improve language modeling performance.

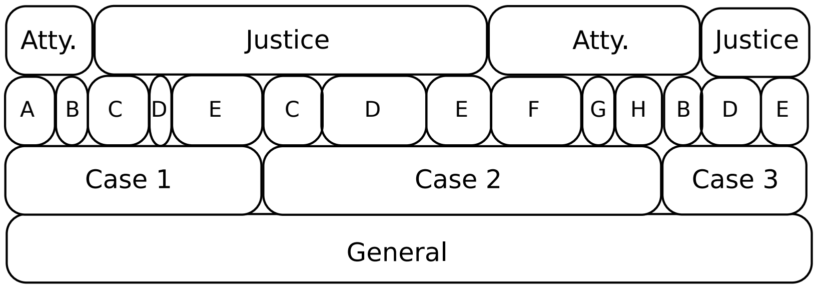
Chapter 5

MODELING OVERLAPPING INFLUENCES WITH A MULTI-FACTOR SLR-LM

The probabilities of word sequences in language are influenced by numerous factors, such as topic, genre, formality, as well as the role, intention and idiosyncrasies of the speaker/author. Furthermore, within a corpus, the scopes of these different influences will vary; for example, in a collection of newswire text the discussion of professional sports is likely to be concentrated in a subset of the documents. These scopes of influence can also be arbitrarily overlapping, as would be the case if you have several speakers/authors covering different sets of topics, or formal and informal examples of language in the form of both written and spoken documents. We illustrate this effect with two hypothetical examples from real corpora. The first is a corpus of spontaneous speech from spoken telephone conversations, where each conversation fits into one of a small set of topics. As shown in Fig. 5.1 (a), two influences are active at a given time: one topic-dependent and one topic-independent (corpus-wide). The second example is a corpus of Supreme Court cases, where the case, the speaker and his or her role in the case is known. This more complicated setup is illustrated in Fig. 5.1 (b). The speaker, the court case and the role of the speaker all augment language-wide factors in an overlapping fashion. As an illustration of how these factors influence the words in a document, Fig. 5.2 displays a quote from Supreme Court Justice Breyer, highlighting the influence of the court case (which affects the topical content), the genre, which here is a general, corpus-wide influence because the entire corpus belongs to the same genre (and accounts for the presence of disfluencies), the speaker (which accounts for the use of “all right” - a Breyer idiosyncrasy) and the role (which accounts for the questioning leading with “in your view”). All highlighted words and phrases were among the top 20 characteristic phrases learned for that particular influence on the data described in Sec. 5.3.



(a) Topic and general influences



(b) Role, speaker, case and general influences

Figure 5.1: Two examples of overlapping scopes of influence: topic in conversational telephone speech (a) and several factors in Supreme Court transcripts (b).

"what about a child who i remember in my third grade my teacher who thought it was her job to teach had problems sometimes with discipline and i might talk too much i used to and and uh so uh the teacher would say that's reasoned self discipline you lack it and i'd get a check and you'd get three checks and you get a mark on your report card all right and say stephen that's the third time you now have a mark on your report card all right now she did that in front of the class because she felt that this is the way i keep my class in order and it helps me teach she did the same thing with her grades many of them she did the same thing with attendance by the way we all said here here sometimes present all right in your view are all those things now forbidden by senator buckley's statute that the teacher cannot run her class that way"

Figure 5.2: Example of different influences on the content of an utterance learned on a Supreme Court case, including topic (a case on privacy in education, in red), the genre (spontaneous speech, in blue), the speaker's idiosyncrasies (Stephen Breyer, in green) and role (Supreme Court Justice, in purple). A word is colored if it is one of the top 20 characteristic n-grams learned for the given influence.

In most language models (LMs), different sources of variation are not explicitly accounted for. Instead, training data from different sources are combined in a mixture model, e.g. [105, 68], or via count merging, e.g. [1], or domain adaptation techniques are used to leverage a general language model in the context of limited in-domain training data [8, 27]. More recently, the impact of topic has been explored using non-parametric Bayesian models, e.g. [13, 122, 120], which use a Dirichlet (or other) prior in unsupervised learning of latent topic variables. In [55], a similar approach is used with latent variables for both topic and role. Most of this work has focused on unigram language models for computational reasons, but n-gram variants of the non-parametric Bayesian topic model are described in [131]. While their framework can be generalized to include multiple factors, non-parametric Bayesian approaches have not been widely adopted; they have a relatively high computational cost and their non-parametric nature makes them somewhat more difficult to interpret.

We propose an alternative approach for characterizing different sources of variation in language: a Multi-Factor Sparse Plus Low Rank (SLR) exponential language model.[1] At the base of the model is a low rank component that, as in Chapter 4, induces continuous representations of words and histories to get a smooth model capturing general syntactic-semantic language behavior. Added to that in the parameter space are arbitrarily many factor-dependent sparse components, each specializing in some phenomenon (e.g. capturing the idiosyncrasies of a speaker or topic) which may overlap in different ways with other factors. By regularizing these components to be sparse, we emphasize the most salient differences and discourage overfitting. In this light, each of the factor-dependent components can be seen as an additive correction to a global model. The model provides a flexible framework for adaptation to a new domain: depending on the nature and the extent of the mismatch, some factors can be updated, some kept intact, and others thrown out entirely.

A key feature of the Multi-Factor model is its interpretability: the elements of the sparse factor-dependent components correspond to keywords that represent salient factor-dependent differences. Unlike past work leveraging topic in exponential models [27, 67], identifying topic-related n-gram keywords is a byproduct; no separate pre-processing step

[1]Portions of this chapter appeared previously in [57].

is used to find them. Further, topic characteristics can be learned jointly with other factors such as genre, speaker, or speaker role. With multiple factors accounted for, the keywords are more meaningful. We explore the use of the learned keywords for topic summarization, and show that they can be used to identify salient characteristics of speaker roles and idiosyncrasies of individual speakers. In contrast to [89], where a sparse plus low rank decomposition of word-document matrices was shown to be effective at identifying document keywords, we need no stop word filtering and support arbitrarily many overlapping factors.

5.1 The Multi-Factor Sparse Plus Low Rank LM

In the Multi-Factor SLR-LM, the monolithic sparse component is replaced with a variable number of factor-dependent sparse components. At any given point in the document, only a subset of all sparse components will be "active." In the example of Fig. 5.1 (a), each n-gram will be associated with a set of three matrices: the (general) low rank component L, the (general) sparse component S_0, and a topic-dependent sparse component S_t. The low rank L exists to capture topic-independent linguistic regularities as before; the general sparse S_0 captures topic-independent exceptions (e.g. genre artifacts like "yeah yeah" or common place names like "new york"); the topic-dependent sparse matrices S_t capture topical exceptions (e.g. "black lab" and "pure bred" for the topic "Pets").

Let $\mathcal{C}_i$ denote the set of components active at word token x_i in the document. We refer to the set of word tokens x_i (and corresponding histories h_i) that have the same set $\mathcal{C}_i$ of active components as a "segment." For example, in Fig. 5.1 (a) there are five segments ($t = 1, 2, \ldots, 5$), while in Fig. 5.1 (b) there are fourteen ($t = 1, 2, \ldots, 14$). Let $\mathcal{C}^{(t)}$ denote the shared set $\mathcal{C}_i$ for all word tokens x_i in segment t; i.e., $\mathcal{C}_i = \mathcal{C}^{(t)}$ for all word tokens i in segment t. The sets $\mathcal{C}^{(t)}$ can be equivalently represented in a binary "scope" matrix, K. The rows of K correspond to the sparse components in the model, while the columns correspond to segments. Fig. 5.3 shows the scope matrix for the Supreme Court example of Fig. 5.1 (b), with sparse components for each speaker, for each case, as well as a sparse component for "justices" and one for "attorney." Then the set $\mathcal{C}^{(t)}$ is just the set of rows $\{j : K_{jt} = 1\}$.

The Multi-Factor LM thus consists of a general low rank L, a general sparse S_0, and C

Adv.	1	1							1	1	1	1		
Justice			1	1	1	1	1	1					1	1
A	1													
B		1							1			1		
C			1			1								
D				1			1						1	
E					1			1						1
F										1				
G											1			
*Case*1	1	1	1	1	1									
*Case*2						1	1	1	1	1	1			
*Case*3												1	1	1
General	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Figure 5.3: A binary "scope" matrix K defining which sparse components (rows) are active in which segments of the document (columns). This key corresponds to the example in Fig. 5.1 (b).

additional sparse components (e.g. the other rows in Fig. 5.3). The average log-likelihood $\mathcal{L}$ of the full dataset $\mathcal{X}$ (with N_t word tokens in segment t and $N = \sum_{t=1}^{T} N_t$ overall) is

$$\mathcal{L}(\mathcal{X}; L, S_0, S_1, \ldots, S_C) = \frac{1}{N}\sum_{i=1}^{N} \log P_{\mathcal{C}_i}(x_i|h_i) = \frac{1}{\sum_t N_t}\sum_{t=1}^{T}\sum_{i=1}^{N_t} \log P_{\mathcal{C}^{(t)}}(x_i|h_i), \tag{5.1}$$

$$P_{\mathcal{C}_i}(x|h) = \frac{\exp\left(\psi(x)^T(L + \sum_{c\in\mathcal{C}_i} S_c)\phi(h)\right)}{\sum_{x'} \exp\left(\psi(x')^T(L + \sum_{c\in\mathcal{C}_i} S_c)\phi(h)\right)}. \tag{5.2}$$

Training involves solving a convex optimization problem:

$$\min_{L,S_0,\ldots,S_C} \left(\gamma_0\|L\|_* + \sum_{c=0}^{C} \gamma_{1c}\|S_c\|_1 - \mathcal{L}(\mathcal{X}; L, S_0, \ldots, S_C)\right). \tag{5.3}$$

We can solve this problem using a modified accelerated proximal gradient descent algorithm, a variant of the algorithm used in Chapter 4 (which was based on [123]); the key difference is that sparsifying line searches are performed in parallel for all sparse components, instead of a single sparse component. Due to the fact that proximal operator for all of the sparse matrices decomposes over individual matrices, the same convergence guarantees apply. We assume that the different sparse components cover different subsets of the data; otherwise, the solution may not be unique.

The training algorithm requires computing the gradient of the smooth part of the objective (the last two terms) with respect to each problem variable. These gradients can be computed efficiently in one pass over the data. Let $\nabla_{A_t}\mathcal{L}$ denote the gradient of average log-likelihood with respect to the sum $A_t = L + \sum_{c\in\mathcal{C}^{(t)}} S_c$, then

$$\nabla_{A_t}\mathcal{L} = E_{\hat{P}(x,h)}[\psi(x)\phi(h)^T] - E_{P_{\mathcal{C}^{(t)}}}[\psi(x)\phi(h)^T] \tag{5.4}$$

where $\hat{P}$ is the unnormalized empirical joint distribution of words and histories occurring in segment t. ($\hat{P}$ sums to the number of words in segment t over the number of words in the corpus.) $P_{\mathcal{C}^{(t)}}$ is similarly unnormalized. Then, the gradients of the smooth part of the objective with respect to the sparse components, S_c, are simply

$$\nabla_{S_c} f_{\text{smooth}} = \sum_{\{t:K_{ct}=1\}} \nabla_{A_t}\mathcal{L} + \gamma_2(L + \sum_{j=0}^{C} S_j). \tag{5.5}$$

That is, one can do a single pass from $t = 1, \ldots, T$ and accumulate each of the $\nabla_{S_c} f_{\text{smooth}}$ along the way.

5.2 English Topic Modeling Experiments

We conducted several English topic modeling experiments to measure the Multi-Factor LM, including its performance in terms of perplexity with joint training and adaption scenarios and in terms of the quality of keywords learned in the sparse components.

5.2.1 Data

We use transcripts from conversational telephone speech form the Fisher corpus, which consists of conversations between strangers on 40 pre-assigned topics. We split (by conversation) each topic into training, development and test sets, yielding 5.5M word tokens of training data, 1.9M word tokens of development data, and 2.0M word tokens of test data. The training set size varied among topics, from as small as 24k training tokens to as large as 366k, with an average of 140k words per topic. Our language model vocabulary includes all word types that appeared nine or more times in the training data, for a vocabulary size of 9.7k (all out-of-vocabulary words are mapped to a dedicated OOV token). Due to our motivation to analyze the exceptions learned, we restrict ourselves to bigram language models in all experiments, which are sufficient for most topical keywords.

Topic-Dependent Language Model

We first consider the joint training case, where our training data consists of the first 20 Fisher topics, split by topic, and we evaluate test set perplexities on each of the same 20 topics; specifically, we report the average test set perplexity over all 20 topics. Using a Multi-Factor LM with sparse component topology analogous to that in Fig. 5.1 (a) (but with 20 topics) we trained a joint model on the training set. Parameters γ_0, γ_{1c} and γ_2 were tuned using coarse grid search on the development set (for simplicity we set γ_{1c} equal for all c). The model we use to compute perplexity on test set topic t is the matched topic-dependent model with parameters $L + S_0 + S_t$.

We also consider another common scenario: the training data and evaluation data have some type of mismatch; specifically, we consider topic mismatch. Our training data consists of the same first 20 topics of the Fisher data used before, while we treat each of the next 20 topics (21-40) as new domains. We adapt our Multi-Factor LM to new test topic t' as follows: 1) from the model trained in Sec. 5.2.1, we keep general L and S_0, but discard all training topic-specific models S_t, 2) we parameterize the adapted model with weights $L + S_0 + S_{t'}$, and 3) we estimate the new $S_{t'}$ by solving the following convex adaptation optimization problem

$$\min_{S_{t'}} \quad \gamma_1 \|S_{t'}\|_1 + \frac{\gamma_2}{2}\|S_{t'}\|_F^2 - \mathcal{L}(\mathcal{X}_{t'}; L, S_0, S_{t'}) \tag{5.6}$$

This is solved by a straightforward variant of the proximal gradient algorithm employed used for training the Multi-Factor LM. There are a few points to note about adaptation. First, the low rank L and general sparse S_0 components are preserved, which assumes that they are capturing topic-independent information; this is a reasonable assumption in our case because topic-dependent n-grams ended up in the various S_t, by design. Second, the adapted models are learned independently from other new topics, rather than jointly. Finally, the

	mKN	Multi-Factor LM	$L + S_0$
Joint Training	81.5	**79.7**	93.1
Adaptation	84.6	**83.3**	98.9

Table 5.1: Jointly trained and adapted test set perplexities, averaged over topics 1-20 (joint) or topics 21-40 (adaptation).

adaptation problem is significantly faster than the original model training, because no low rank component is being learned.

As a baseline we compare against an n-gram model with modified-Kneser-Ney (mKN) smoothing; to evaluate topic t we linearly interpolate a general model (trained on the first 20 topics) with a topic-dependent model trained only on topic t's data using the SRILM toolkit [118]. (We found linear interpolation to perform better than count merging for this task.) In the "Joint case," the topic training data is accounted for in the general model, and in the "Adaptation case" it is not. The results are presented in Table 5.1. In perplexity, the Multi-Factor SLR-LM performs similarly to the baseline, slightly edging the modified-Kneser-Ney interpolated models by 2%. In the last column we see that the perplexities using models parameterized by $L + S_0$ only (i.e. omitting the topic-dependent factors) are much worse, suggesting that the topic-dependent factors play a prominent role in capturing the language behavior.

Keyword Extraction

Apart from its role as a language model, the Multi-Factor SLR-LM is of interest for its ability to identifying keywords associated with the factors. Specifically, the sparse entries of the S_t components contain the corrections to the general model for the factor-specific case; that is, they distill out the key differences between general and factor-dependent language. Fig. 5.4 illustrates this, displaying the top 30 non-zero elements (by weight) in a sparse topic-dependent matrix. From the entries, it is clear that this matrix is finding words and phrases that are particularly characteristic of the topic of sports. We use this behavior as a mechanism to automatically identify topic keywords.

Example Sparse Entries

super bowl	green bay	figure skating	spring training	world cup
world series	die hard	winter olympics	sporting event	summer olympics
baseball	season tickets	football	soccer	car racing
sport	hockey	basketball	e s	p n
sports	watch	minor league	golf	final four
teams	s p	olympics	professional	stadium

Figure 5.4: Top 30 non-zero elements of a topic-dependent sparse matrix (left to right, then top to bottom); in this case, on the topic of Sports.

In this section we evaluate the effectiveness of this approach to keyword extraction; to measure quality, we collect the largest positive magnitude 30 entries in each of the 20 learned topic-dependent sparse matrices and use those as candidate keywords. Recall that our "keywords" can be any order of n-gram; because our model in Sec. 5.2.1 is a bigram, the keywords learned here are unigrams and bigrams.

We compare against two other keyword extraction methods, which each make use of a special word-document matrix (technically n-gram-topic matrix - the rows are all bigrams and unigrams observed in the data and the columns are the 20 topics). The first baseline reweights the matrix using the standard term-frequency inverse-document-frequency (TF-IDF) scheme; after reweighting, the largest 30 entries in each column are used as the keywords. In our second baseline, inspired by feature selection, we use mutual information (MI) between the features (n-grams) and the topics (binary one-vs-rest) to rank the features per topic; the top 30 largest n-grams per topic after stop word filtering are selected as keywords.[2] (In contrast, the Multi-Factor and TF-IDF methods did not require any stop word filtering.)

The keywords from all three methods were combined, with order randomized, and labeled as clearly-topically-relevant or not by two annotators unaffiliated with this research. Fig. 5.5

[2]We use a custom stop word list containing 280 stop-words, compiled by Amittai Axelrod of the University of Washington (see Appendix A).

plots the percentage of keywords that were rated as clearly-relevant (averaged over 20 topics and two annotators), in three bins: the top 10 rated keywords per topic, keywords 11-20, and keywords 21-30. While the Multi-Factor model has the highest percentage of relevant keywords at each level, the biggest gains are due to the quality of keywords decaying more slowly in Multi-Factor model than the other baselines. Over all bins, the precision of keywords learned by the Multi-Factor model are 13% absolute better than the TF-IDF method and 31% absolute better than the mutual information approach.

In Tab. 5.2, we display the top three unique candidate keywords for each method that were deemed to be valid keywords by the annotators, with the goal of illustrating the relative strengths of each method. The Multi-Factor method finds a diverse set of "colorful" keywords, the MI method finds "plain" keywords, and the TF-IDF method finds keywords somewhere in-between. We believe it is the diversity of the Multi-Factor's lists that allow it to perform better, particularly as one travels further down the keyword lists.

We compare against TF-IDF and mutual information because they are standard, easy-to-use techniques. However, researchers have introduced more sophisticated models for finding topically relevant words and phrases. In [129], Wang et al. introduce the Topical n-Gram Model (TNG), a hierarchical Bayesian model similar to Latent Dirichlet Allocation (LDA) [14]. Like our approach, the TNG is able to learn topical-dependent unigrams and bigrams. Exact inference is intractable for the TNG, necessitating the use of approximate inference techniques. Unlike our approach, however, the TNG does not need supervised topic labels, which is convenient when no such labels are available beforehand. In [64], Johnson draws the connection between LDA and probabilistic context free grammars, and uses this to motivate new topic models that combine attributes of both. No formal evaluation is provided of their topic model that support topical n-grams, but illustrative examples suggest that it is capable of learning good topic-dependent n-grams. Like the TNG, no topic labels are needed.

5.3 English Supreme Court Modeling Experiments

We also informally investigate the modeling of overlapping factors in Supreme Court transcription, with an overlapping factor configuration similar to that shown in Fig. 5.1 (b) (but with more cases and speakers).

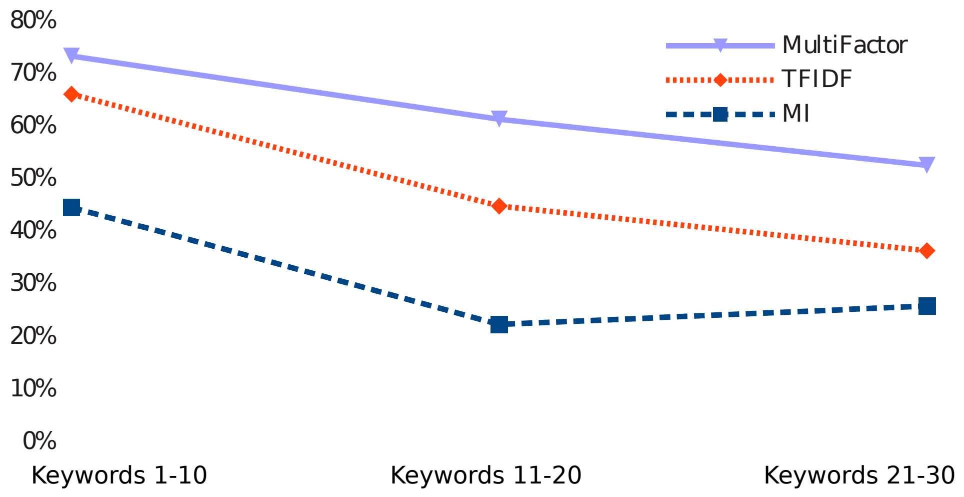


Figure 5.5: Percentage of keywords labeled as relevant, for the Multi-Factor, TF-IDF and Mutual Information methods. Results averaged over 20 topics and two annotators.

Top Unique Keywords, By Method

	"Life Partners"	"Minimum Wage"	"Sports"
Multi-Factor	soul mate	food stamps	super bowl
	problem solving	flipping burgers	spring training
	physical attraction	poverty line	winter olympics
TF-IDF	compatibility	living wage	skating
	get married		the olympics
	humor		playoffs
MI	life	dollars	watching
	years	pay	
	together	work	

Table 5.2: Top three unique keywords for each method, highlighting the strengths of each. When fewer than three keywords are listed, it is because the method did not generate three unique candidate keywords rated as actual keywords by the annotators.

5.3.1 Data and Model

We use a 20 court case transcript subset of the Supreme Court corpus.[3] Since these experiments only involve a qualitative discussion, we use a single (training) set, with 207k word tokens (7.3k word types) from 58 speakers. All speakers are labeled with one of two roles: justice or attorney. We again use bigrams, and our vocabulary consists of all 7.3k word types that appeared in the training data.

5.3.2 Discussion

In our trained model, we find topically relevant keywords appearing for the case-dependent sparse matrices. Examples include:

1. **Rush Prudential HMO, Inc. v. Moran**. An HMO denying a request to cover a surgery: "savings clause," "medical necessity," "h m," "m o," "pilot life."

2. **TRW v. Andrews**. Allegations of violating the Fair Credit Reporting Act: "equitable estoppel," "reporting agency," "misrepresentation exception," "liability arises."

3. **Harris v. United States**. Regarding the sale of illegal narcotics while carrying an unconcealed firearm: "mandatory minimum," "reasonable doubt," "seven years."

4. **Edelman v. Lynchburg College**. A claim of discrimination in the denial of tenure at a college: "under oath," "title VII," "common law."

5. **Owasso Independent School Dist. No. 1011 v. Falvo**. A claim that peer grading violates FERPA: "school district," "tenth circuit," "grade book," "education record."

6. **Toyota Motor Mfg v. Williams**. A claim of assembly line work leading to carpal tunnel syndrome: "worker's compensation," "assembly line," "life activity."

[3]www.oyez.org

The model also learned characteristic language associated with the roles of justice, including question words ("why," "how,") and confirmations structured as statements ("you're saying," "your view," "I thought"), and attorney, including deferential language ("your honor," "chief justice," "that's correct") and hedging ("I think").

The per-speaker factors are most reliable for justices, for whom the data covers several cases. In this, we captured speaker idiosyncrasies (e.g. Breyer's habit of starting sentences with "all right" and Scalia's disfluencies) and Rehnquist's expressions that are characteristic of the role of Chief Justice (e.g. "we'll hear," "minutes remaining," "is submitted").

5.4 *English Genre Modeling Experiments*

In Sec. 5.2 we showed that the Multi-Factor model can account for variation in topic, and in Sec. 5.3 we saw that it can account for the overlapping influences of topic, speaker and role, but in both cases the our data came from a single genre (telephone conversations and court cases, respectively). In this section we take a brief look into how the Multi-Factor model handles data coming from heterogeneous genres. We revisit the two television genres encountered in experiments of Chapter 3, using a small amount of data from the "broadcast conversations" (BC) genre, which are political and news talk shows that are largely unscripted discussions or interviews between hosts and guests, and a larger amount of the more abundant broadcast news (BN) genre, which is mostly scripted and more formal in tone. The data and vocabulary configuration is the same as in Chapter 3 (the statistics are repeated for convenience in Tab 5.3). Our vocabulary size is 5k and all OOV tokens are mapped to a dedicated OOV token. Our Multi-Factor model has four weight matrices: a general low rank L, a general sparse S, a BC-dependent sparse S_{BC} and a BN-dependent sparse S_{BN}.

After jointly training, we evaluate on the BC Test set using weights $L + S + S_{BC}$ (discarding the BN-dependent weights). The results are listed in Tab. 5.4. In contrast to the interpolated mKN bigram models trained on BC and BN, which uses a held-out development set to find the optimal interpolation weights, the Multi-Factor has no such source of supervision guiding the relative importance of the two data sources. Despite this limitation, it recovers a fair amount of the perplexity obtained by incorporating the BN

Dataset	Size (words)
BN Train	3.2M
BC Train	99K
BC Dev	189K
BC Test	136K

Table 5.3: Multi-Factor genre modeling experiment data.

Model	Test Set Perplexity
mKN BC-only	166.7
Multi-Factor ($L + S + S_{BC}$)	118.5
mKN BC + mKN BN	82.6

Table 5.4: Multi-Factor genre modeling experiment results.

data. Ideally, the method would be able to permit more control over the relative importance the model gives to different portions of the data to permit it to beat the 82.6 perplexity number. Extensions to permit control over the relative importance of particular subsets of the data is an important future direction.

In Tab. 5.5, we take a look at *what* the S_{BC} and S_{BN} matrices are learning. The top 15 positive exceptional bigrams are listed for BC and BN. The initial observation is that it appears to be modeling the topical differences between the two data associated with the two genres more so than the actual genre differences themselves. However, some genre phenomena do appear; for example, the presence of "the" in the BN list does appear to be capturing a true genre difference, since the relative frequency of the word "the" is roughly 10% lower in BC than BN.

In Tab. 5.6 we list the exceptions we deem to be indicative of spontaneous, conversational speech among the top 200 exceptions for BC and for BN. In this case, more genre differences are evident. BC shows, which often involve discussion of personal positions on issues, include phrases reflective of this interaction, such as "ya know," "i wanna," and "yea."

Given the frequency of contention in BC shows, it is not surprising that they also show disfluencies associated with holding the floor, like "um" and "uh." Whereas in BC, with mostly in-studio discussions where nodding and other non-verbal back-channels are possible, the interaction between in-studio anchors and remote reporters in BN is more similar to telephone interaction, with verbal backchannels like "uh-huh" and phrases like "bye bye" at the end of conversations.

Despite this evidence that some genre phenomena are being captured, the emphasis of topical phrases in the lists suggests a straightforward extension of the Multi-Factor model, where one is not limited to modeling the influence of different factors as sparse corrections, but as low rank corrections (instead or in addition to the sparse corrections). Such a structure would likely be better able to handle the systematic rather than sparse nature of language usage differences between genres. While this extension can be handled with minor variants of modeling framework and training algorithms already introduced, we leave the assessment of such an extension to future work.

5.5 Conclusions

In summary, we introduced a multi-factor exponential language model that allows supervised learning of overlapping factors that influence sequential language behavior. As a language model, the model provides small gains in perplexity in a topic adaptation scenario compared to a baseline modified-Kneser-Ney that interpolates general and topic specific models. It can also be used as a mechanism to identify factor-dependent characteristics. In particular, the n-gram elements encoded in sparse parameter matrices give an intuitive way to identify factor-dependent keyword phrases. On a conversational speech task, we demonstrate that human raters prefer topic keywords learned by the multi-factor model over TF-IDF and mutual information baselines. With Supreme Court transcripts, we show qualitatively the ability to learn factor-dependent keywords for different court cases, roles (justice vs attorney), and speakers. Experiments modeling heterogeneous genres show some promising initial perplexity results, but suggest that low-rank corrections may be useful in addition to (or possibly instead of) the sparse correction structure when the differences are systematic rather than exceptional. In addition to summarization, identification of keyword

BC Exceptions	**BN Exceptions**
san francisco	los angeles
hum v's	et cetera
thelma wahl	al qaeda
neil entwistle	bin laden
al queda	bin laden's
mike gowan	abu ghraib
leslie ballin	the
patricia whitfield	abu musab
robert bork	cnn com
chicken hawk	dianne feinstein
mary kero	sharm el
bob woodruf	al qaida
representative kilmartin	aneesh raman
micah garen	keith oppenheim
emile lahoud	sherrod brown

Table 5.5: Top 15 bigram exceptions in the BC and BN matrices.

BC Spontaneous Exceptions	**BN Spontaneous Exceptions**
ya know	ha ha
mhm	uh huh
i wanna	bye bye
uh	whoa whoa
eh eh	
lets lets	
yea	
um	

Table 5.6: Bigram exceptions indicative of spontaneous, conversational speech data found among the top 200 BC and BN exceptions.

phrases is of interest for feature selection, learning lexical items (as shown in Chapter 4), and detecting new events by learning keywords on new data sources. Encoding words with other features (e.g. morphological structure, syntactic dependents) would also make it possible to identify other types of idiosyncratic phenomena.

Chapter 6

MODEL EXTENSIONS

This thesis has focused on the task of modeling the probability of a sequence of categorical symbols, $p(x_1, x_2, \ldots, x_T)$; in particular, on the language modeling case where the symbols are words. We also explored how byproducts of our model, such as sparse matrix entries, could be useful for other language processing tasks. However, the low rank ideas developed here can be easily translated to other modeling problems. In this chapter we will introduce the models and algorithms for two important extensions: first, to the sequence tagging problem and second to the acoustic modeling problem. The goal here is to show that the techniques developed are broader than language modeling; we leave experimental evaluation to future work.

6.1 Sequence Tagging

The problem of *sequence tagging* is closely related to the language modeling problem studied extensively in this thesis. In tagging, there are two parallel sequences: $x_1, \ldots, x_T$ and $y_1, \ldots, y_T$. For example, x_i may denote the i-th character in a word, while y_i denotes its corresponding pronunciation (possibly null if not pronounced). Often the goal is to predict the sequence of labels $y_1, \ldots, y_T$ from some sequence of observations $x_1, \ldots, x_T$. This can be accomplished either by modeling the joint distribution $P(x_1, \ldots, x_T, y_1, \ldots, y_T)$, or by modeling the conditional distribution $p(y_1, \ldots, y_T | x_1, \ldots, x_T) = p(\mathbf{y}|\mathbf{x})$ directly, where we will use bold lowercase letters to denote sequences. Clearly, this is a more challenging task: there are potential interactions between y's, between x's, and between y's and x's. As in language modeling, conditional independence assumptions are often made to control model complexity.

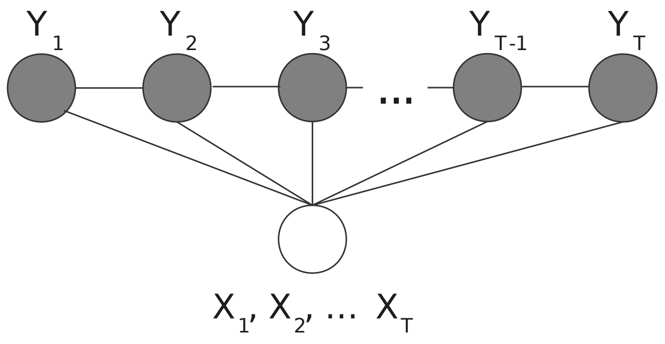


Figure 6.1: A graphical representation of the conditional independence assumptions in the first-order linear chain conditional random field.

6.1.1 The Low Rank Conditional Random Field

Two different models are popular in the field of language processing for sequence tagging: hidden Markov models and conditional random fields (CRFs). The feature-based approach of the exponential models introduced in Chapter 4 is most naturally incorporated into the CRF, so it will be our focus. If we assume that the label sequence is a first-order chain, we obtain the *linear chain CRF*, which is defined to be:

$$p(\mathbf{y}|\mathbf{x}) = \frac{\prod_{i=1}^{n} \exp(\theta^T f(y_{i-1}, y_i, x))}{\sum_{\mathbf{y}'} \prod_{i=1}^{n} \exp(\theta^T f(y'_{i-1}, y'_i, x))}. \tag{6.1}$$

The conditional independence assumptions of this undirected graph are shown in Fig. 6.1; the basic assumption is that the probability can be decomposed into potentials over pairs of adjacent words and the entire input sequence $\mathbf{x}$. Feature function f maps to $\mathbb{R}^d$, and the model is parameterized by a weight vector $\theta \in \mathbb{R}^d$. In a higher-order (nth-order) linear chain CRF, is

$$p(\mathbf{y}|\mathbf{x}) = \frac{\prod_{i=1}^{n} \exp(\theta^T f(y_{i-n}, \dots, y_i, x))}{\sum_{\mathbf{y}'} \prod_{i=1}^{n} \exp(\theta^T f(y'_{i-n}, \dots, y'_i, x))}. \tag{6.2}$$

The primary complication of moving to larger orders is in inference, which is needed both to make predictions and during parameter estimation. Exact inference is quadratic in the size of the label alphabet for linear chain CRFs, and quartic in the size of the label alphabet for second-order chain CRFs. We focus our discussion on first order models for notational simplicity.

Continuing the feature-based approach introduced in Chapter 4, we define a function Υ that maps symbols y_i to a feature representation in $\mathbb{R}^{d_\Upsilon}$ and a function χ that maps input sequence $\mathbf{x}$ to a feature representation in $\mathbb{R}^{d_\chi}$. A common simplifying assumption is that features are only extracted from the ith element of sequence $\mathbf{x}$, so with some abuse of notation $\Upsilon(\mathbf{x}) = \Upsilon(x_i)$. Because the dimension of the label alphabet in most natural language processing tasks is typically moderate to small, d_Υ will not be very large. If there is no natural feature representation of the labels, Υ can simply map to the one-hot (indicator) encoding of the label.

Matrix Version

Making the assumption that only the ith element of $\mathbf{x}$ is used, the linear chain CRF has a feature function $f(y_{i-1}, y_i, x_i)$, which is often broken into two components, each with its own set of weights:

$$p(\mathbf{y}|\mathbf{x}) = \frac{\prod_{i=1}^{n} \exp(\theta_1^T f(y_{i-1}, y_i) + \theta_2^T f(y_i, x_i))}{\sum_{\mathbf{y}'} \prod_{i=1}^{n} \exp(\theta_1^T f(y'_{i-1}, y'_i) + \theta_2^T f(y'_i, x_i))}. \tag{6.3}$$

Introducing the feature functions, this can be modified to capture bilinear relationships:

$$p(\mathbf{y}|\mathbf{x}) = \frac{1}{Z(\mathbf{x})} \prod_{i=1}^{n} \exp\left(\Upsilon(y_{i-1})^T A_1 \Upsilon(y_i) + \Upsilon(y_i)^T A_2 \chi(x_i)\right). \tag{6.4}$$

A_1 and A_2 are weight matrices. The training objective is

$$\max_{A_1, A_2} LL(\mathcal{X}; A_1, A_2) - \mu_1 \|A_1\|_* - \mu_2 \|A_2\|_*. \tag{6.5}$$

This is similar to the standard nuclear norm regularized problem, and can be solved with an accelerated proximal gradient algorithm [123], as was used in previous chapters. The cost of computing the gradient is increased over the language modeling case, because a dynamic programming algorithm must be run for each training point in order to compute the feature expectations, despite the existence of supervised training data.

As expected, learning low rank A_1 and A_2 has the effect of inducing low dimensional representations of tag labels (e.g. part-of-speech tags) and input sequence tokens (e.g. words). However, note that low dimensional representations learned are not coupled. If the optimal $A_1 = U_1V_1^T$ and $A_2 = U_2V_2^T$, then the three maps from label sequence $\mathbf{y}$ to low-dimensional representations, U_1^T, V_1^T and U_2^T, need not be the same. Indeed, U_1^T and U_2^T need not map to the same lower dimension. This is (at least partially) desirable behavior. If U_1^T and V_1^T were coupled, it would favor sequences that repeated labels. Constraining U_1^T to be near U_2^T assumes that the same mapping preserves the discriminative information for both potential functions (between pairs of labels and between labels and input elements), and that the optimal lower dimension is the same, either of which may or may not be true. Depending on the nature of the problem, it may also be of interest to frame the weights as sparse plus low rank, so that $A_i = S_i + L_i$. The problem remains convex, and could be handled using the same optimization techniques applied in Chapters 4 and 5.

Tensor Version

While the matrix variant captures two pair-wise relationships with two bilinear structures (matrices), one can also directly capture multi-way relationships directly with a multilinear structure (tensor). There are a few natural ways to approach this. First, in the linear chain CRF discussed above, the function $f(y_{i-1}, y_i, x)$ need not be broken into two, but instead modeled multilinearly. This gives a model of the form

$$p(\mathbf{y}|\mathbf{x}) = \frac{1}{Z(\mathbf{x})} \prod_{i=1}^{n} \exp\left(\mathcal{A} \times_1 \Upsilon(y_{i-1}) \times_2 \Upsilon(y_i) \times_3 \chi(x)\right). \tag{6.6}$$

Recall from Chapter 2 that tensor multiplication generalizes matrix multiplication: $\mathcal{A} \times_i z$ denotes multiplying vector z along the i-th mode of a tensor $\mathcal{A}$. Generalizing the above model to an m-order Markov assumption on the labels yields

$$p(\mathbf{y}|\mathbf{x}) = \frac{1}{Z(\mathbf{x})} \prod_{i=1}^{n} \exp\left(\mathcal{A} \times_1 \Upsilon(y_{i-m}) \times_2 \Upsilon(y_{i-m+1}) \times_3 \cdots \times_{m+1} \Upsilon(y_i) \times_{m+2} \chi(x)\right). \tag{6.7}$$

Either of the above models can be solved with the CMLE algorithm [112], optimizing the non-smooth, convex objective:

$$\min_{\mathcal{A}} \|\mathcal{A}\|_* - LL(\mathcal{X}; \mathcal{A}). \tag{6.8}$$

Because this approach has greater time complexity than the problems discussed in previous chapters, steps must be taken to keep the feature dimensions and tensor order down (e.g., by projecting large feature representations to lower dimensional space before training the model).

6.1.2 Summary

In this thesis, we have seen how sequences of symbols (typically words) can be modeled with bilinear and multilinear models, and how the notions of matrix rank and sparsity can give rise to models that are both easy to train and easy to interpret. The sequence tagging problem takes this one step further: again we are determining probabilities of a sequence $\mathbf{y}$, but in this case we also condition on a separate sequence $\mathbf{x}$ whose information is critical to determining $\mathbf{y}$'s probability. Compared to the language model introduced in Chapter 4, the conditional random field discussed here has two key differences. First, we need to normalize over entire sequences, rather than just the next symbol. This only affects the log-probability and can be addressed with dynamic programming in a standard way. The second is that the need arises to model general multilinear relationships (as opposed to the history-word bilinear relationship). This is accomplished by replacing matrix weights with tensor weights, and replacing the matrix nuclear norm to encourage low matrix rank with the tensor nuclear norm to encourage low tensor n-rank. Like a low rank matrix, a low n-rank tensor induces low-dimensional representations of the objects it is modeling. Conveniently, training these proposed tagging models involves solving convex optimization problems for which suitable algorithms already exist.

6.2 Acoustic Modeling

An *acoustic model* describes the probabilistic relationship $P(o_1, \ldots, o_T | s_1, \ldots, s_T)$ between an acoustic observation sequence $o_1, \ldots, o_T$ and the corresponding latent linguistic state sequence $s_1, \ldots, s_T$ that produced it.[1] We focus in particular on the state-dependent emission distributions, $p(o|s)$, which are typically Gaussian mixture models. Each state models the acoustic characteristics of a portion of a context-dependent phone. Research has shown

[1]Typically in speech recognition the problem is phrased as giving the probability of acoustic observation given a word sequence, but ultimately the word sequence must be mapped to a state sequence; e.g., with a pronunciation dictionary mapping words to a sequence of individual context-dependent phone models.

that context-dependency improves performance because it better models phenomena like coarticulation, but there are limits to the context that can be accounted for. With a phonetic inventory of 50 and three states per context-dependent phone, there are 375k possible triphones or 937.5M possible quinphones. Clearly, no reasonable amount of training data supports training such a large number of context-dependent models. Training each of these models can be seen as solving related sub-problems, which suggests that we should solve them jointly. The dominant strategy of explicitly tying models using a decision tree which partitions the tri- or quinphone set by asking questions about the left and right context is interpretable and effective, but it relies on knowledge of the target language for designing possible questions. A more recent technique for training models jointly is known as the Subspace Gaussian Mixture Model [97], in which a large set of mixtures are shared among all states, but the mean vectors are distinct for each state, living in a subspace of possible mean vectors. We propose here a general, principled approach, where shared, continuous representation of phones and contexts are learned and used to solve each of the individual context-dependent phone modeling problems jointly.

Specifically, we will propose a new approach to acoustic modeling, in which multilinear relationships between the acoustics, phone, context and states are learned, and the complexity is controlled by the n-rank of a weight tensor. This not only gives us control over model complexity, but as a byproduct learns a low-dimensional representation of the acoustics, the phone, the context and the state. It then is capable of defining distinct (but multilinearly related) distributions for *all* possible tri- or quin-phone states. We will show that it can be used directly as an acoustic model, as a mechanism to learn feature transformations and as a way to partition the space of context-dependent phones, allowing it to be used as a clustering method in traditional acoustic models.

6.2.1 The Low n-Rank Tensor Acoustic Model

The dominant approach to speech recognition uses a Hidden Markov model with Gaussian mixture models for the output distributions. However, one can plug in a different output distribution; this is commonly referred to as a “hybrid” approach. Hybrid approaches typically involve a discriminative state posterior model $p(s|o)$ and Bayes' rule:

$$p(o|s) = \frac{p(s|o)p(o)}{p(s)} \tag{6.9}$$

The maximum likelihood estimate of the denominator $p(s)$ can be obtained trivially from an initial alignment. The $p(o)$ term can be ignored during decoding (i.e. finding the most probable state sequence), since it is a non-negative constant. Thus the focus is on the state posterior model, $p(s|o)$.

Although neural networks are the most famous example for $p(s|o)$ in hybrid automatic speech recognition (ASR), one can use any model that produces valid probabilities, including a maximum entropy model. In this case, the state posterior probabilities are given by:

$$p(s|a) = \frac{\exp(w_s^T a)}{\sum_{s'} \exp(w_{s'}^T a)}. \tag{6.10}$$

Here $s \in \{1, 2, \ldots, C\}$ is the state index, $w_s \in \mathbb{R}^D$ is a state-dependent weight vector and $a \in \mathbb{R}^D$ is the acoustic feature vector. An example of an approach similar to this is presented in [52], where phone classification performance was comparable to many published results, but not state-of-the-art. One obvious way to extend the above model is to incorporate second order features, replacing the state-dependent weight vectors with weight matrices:

$$p(s|a) = \frac{\exp(a^T W_s a)}{\sum_{s'} \exp(a^T W_{s'} a)}. \tag{6.11}$$

This is equivalent to Eqn. 6.10 where a contains all pairs of products between the frame's acoustic feature elements (along with the original features themselves), and is a standard way to improve linear discriminability between classes. Note that our overall set of parameters is a tensor $\mathcal{W}$, where each W_s is just a matrix slice of $\mathcal{W}$. If we let z denote an indicator vector of the state s, we get the same equations in a different form:

$$p(s|a) = \frac{\exp(\mathcal{W} \times_1 a \times_2 a \times_3 z)}{\sum_{z'} \exp(\mathcal{W} \times_1 a \times_2 a \times_3 z')} = \frac{\exp(\sum_{i,j,k} a_i a_j z_k \mathcal{W}_{ijk})}{\sum_{z'} \exp(\sum_{i,j} a_i a_j z' \mathcal{W}_{ijk})}. \tag{6.12}$$

The problem with this parameterization is that there can be too many free parameters, and there is no inherent tying between states. This is remedied by imposing a structure on $\mathcal{W}$; namely, making it have low n-rank. Recall that the n-rank of a tensor is a tuple of scalars, one for each mode. If $\mathcal{W}$ has n-rank $< r_1, r_2, r_3 >$, then there exist matrices $U_1 \in \mathbb{R}^{D\times r_1}, U_2 \in \mathbb{R}^{D\times r_2}$ and $U_3 \in \mathbb{R}^{C\times r_3}$ and a core tensor $\mathcal{G} \in \mathbb{R}^{r_1\times r_2\times r_3}$ such that

$$p(s|a) = \frac{\exp(\mathcal{W} \times_1 a \times_2 a \times_3 z)}{\sum_{z'} \exp(\mathcal{W} \times_1 a \times_2 a \times_3 z')} = \frac{\exp(\mathcal{G} \times_1 (U_1^T a) \times_2 (U_2^T a) \times_3 (U_3^T z))}{\sum_{z'} \exp(\mathcal{G} \times_1 (U_1^T a) \times_2 (U_2^T a) \times_3 (U_3^T z'))}. \tag{6.13}$$

This has the following interpretation: rather than model the second order features of the input directly, we first learn two linear transformations of the acoustic features, U_1 and U_2 that project a down to r_1 and r_2 dimensional representations, and then model the second order features of those lower-dimensional features (preliminary experiments suggest that, as expected, $U_1 = U_2$ due to the symmetry of the problem setup). Further, we learn a low dimensional representation of the *state*, $U_3^T z$, so the weight matrix for state s is a linear combination of r_3 basis matrices ($U_3^T z$ are the mixture weights). This gives a mechanism to exploit similarities between states.

We can go even further, though, by using a factored representation of the state. Rather than have a single state indicator vector z, say we could have three indicator functions, $\Phi^C(s)$, $\Phi^L(s)$ and $\Phi^R(s)$ that map a state s to its center phone identity and left and right phone contexts, respectively. If we let $\mathcal{W}$ be the corresponding fifth-order tensor, then we obtain the following acoustic model:

$$\begin{aligned} p(s|a) &= \frac{\exp(\mathcal{W} \times_1 a \times_2 a \times_3 \Phi(s)^C \times_4 \Phi^L(s) \times_5 \Phi^R(s))}{\sum_s \exp(\mathcal{W} \times_1 a \times_2 a \times_3 \Phi^C(s') \times_4 \Phi^L(s') \times_5 \Phi^R(s'))} && (6.14) \\ &= \frac{\exp(\sum_{i,j,k,\ell,m} a_i a_j \Phi_k^C(s)\Phi_\ell^L(s)\Phi_m^R(s)\mathcal{W}_{ijk\ell m})}{\sum_{s'} \exp(\sum_{i,j,k,\ell,m} a_i a_j \Phi_k^C(s')\Phi_\ell^L(s')\Phi_m^R(s')\mathcal{W}_{ijk\ell m})}. && (6.15) \end{aligned}$$

Assuming the state labels are obtained via a forced alignment from an HMM-GMM system, there will be a limited number of state labels, so the sum in the denominator of Eqn. 6.15 is over the same set of states as in Eqn. 6.13. But when we regularize $\mathcal{W}$ to be low weight we now learn five basis matrices, $U_1, U_2, \ldots, U_5$. As before U_1 and U_2 are transformations of the acoustic features a (and should be equal), while U_3 now gives a low dimensional representation of the center phone, U_4 gives a low dimensional representation of the left context phone and U_5 gives a low dimensional representation of the right context phone. For notational simplicity we do not explicitly include the phonetic state index, or other potentially useful factors like tone in tonal languages. Those can be included either by expanding the order of the tensor, or by creating distinct indicator entries for different state or tone labels. Quinphone models are a straightforward generalization of triphone models. Factoring the representation has a few key benefits:

- The weight tensor allows all possible triphones to be covered, and due to the smoothing effect of the n-rank regularization, the model can will learn non-zero probabilities for all triphones. For practical speed considerations during training, one may restrict the support to cover only the triphones observed in the training data and normalize over only this subset of triphones. At run-time, any set of triphones can be accommodated simply by changing the normalization term.

- We can use the low dimensional representations to cluster the center phone and left and right context phones, yielding a similar effect as decision tree state tying. This allows the weights learned in our model to be injected back into a standard HMM-GMM, which may be desirable for practical reasons (e.g. using standard, well-optimized tools). It may also benefit other discriminative acoustic models (e.g. deep neural networks), whose triphone state inventory typically comes from a baseline HMM-GMM system. Earlier work [37] showed clearly that using context-dependent phone states as opposed to context-independent ones improves deep neural network acoustic modeling performance; it is possible that better triphone state sets could provide additional benefit.

6.2.2 Training

With or without a factored representation of the state, the model can be trained by solving a non-smooth convex optimization problem:

$$\min_{\mathcal{W}} \|\mathcal{W}\|_* - LL(\mathcal{X};\mathcal{W}). \tag{6.16}$$

LL is the average log-likelihood of the data ($\frac{1}{N}\sum_{i=1}^{N} p(s_i|a_i)$) and $\|\cdot\|_*$ denotes the convex tensor nuclear norm. It can be solved using the Convex Multilinear Estimation algorithm [112].

6.2.3 Computational Complexity

The primary limitations of this approach are the time complexity (one singular value decomposition is required for each mode of the tensor at each iteration) and the memory complexity (one needs to be able to hold a full-rank, dense tensor in memory). To give a sense of the computational costs, we consider two scenarios with typical placeholder values, the first does not factor state representation, while the second does.

- *Non-factored state.* If we use the non-factored triphones, with 5000 triphone phone states, and our acoustic features are the second order features of the concatenation of seven frames of 13-dimensional mel frequency cepstral coefficients, the tensor contains $5000(13 \cdot 7)^2 \approx 41$M elements and uses roughly 0.33 GB of memory (assuming double precision). Three SVDs would need to be computed at each iteration (two acoustic feature modes and one state label mode), two on matrices of dimension 91×455000 and one on a matrix of dimension 5000×8281. The first would be fairly fast and the last not prohibitive (we have no trouble with matrices larger than that in the SLR-LM).

- *Factored state.* Using the same second order acoustic features, but instead using a factored representation of the triphone state, with a phone inventory of 40 and 3 states per phone, the tensor would contain $3 \cdot 40^3(13 \cdot 7)^2 \approx 1.6$B elements, and require 12.7GB to store. Now six SVDs would be required per iteration (one state mode, three phone modes and two acoustic feature modes). The first matrix would be dimension 3×530M, the next three dimension 40×39.7M and last two dimension 91×17.5M. Due to the extreme asymmetry in the dimensions, the SVD computation costs are surprising light. (Recall that the SVD has linear cost in the larger dimension and quadratic cost in the smaller dimension).

6.2.4 Conclusions

Acoustic modeling is a challenging problem that requires trade-offs between accuracy and trainability. The literature clearly indicates that context-dependent modeling improves performance, but this comes at a cost. If there are K phones, then a context-dependent acoustic model is tasked with assigning an emission distribution to K^3 triphones (or K^5 quinphones). Viewing the training of each context-dependent model as a sub-problem to solve, one observes that there are many ways in which the sub-problems are related. Intuitively, context-dependent phones sharing a common center phone should have some underlying commonalities, but to a lesser extent, so should those sharing the same left or right contexts. The traditional strategy of clustering context-dependent phones can be viewed (like n-gram back-off) as a rigid mechanism for solving related problems jointly. In that case, sub-problems are either merged entirely or left completely distinct. We propose here a a more flexible way to exploit the commonalities, by phrasing the training problem as estimating a low n-rank tensor which learns a shared representation of the acoustics and the state. Conveniently, the joint training problem is convex, and algorithms exist to solve it.

Besides providing state posterior probabilities for use in hybrid ASR, our model has other potential applications. The low-dimensional representations it learns can be used to cluster triphones (or quinphones), which can improve traditional HMM-GMM systems (at the least, it removes the need for linguistic knowledge when determining state tying), but can also provide better targets for other state-of-the-art discriminative acoustic models (e.g. deep neural nets) whose triphone state inventory often comes from a baseline HMM-GMM. It is completely compatible with the "tandem" ASR approach, whereby state posteriors are used as features (typically after another dimensionality reducing transformation). Finally, the acoustic feature transformations learned by the model may be useful themselves as a data pre-processing stage, since by design they preserve information that is discriminative when a model makes use of their second order features (as is the case in a full covariance GMM).

Chapter 7
SUMMARY AND FUTURE DIRECTIONS

7.1 Summary of Contributions

7.1.1 New Modeling Frameworks and Experimental Findings

Many fundamental language processing problems have a vast ambient dimension, requiring the use of simplifying assumptions when building models of language. The primary example in this thesis is the assigning probabilities to sequences of categorical variables, $p(x_1, x_2, \ldots, x_T)$, whose joint probability table has v^T entries (where v is the size of the vocabulary from which the variables are drawn). Standard non-parametric maximum likelihood estimates of this table are plagued with zero probability estimates, and thus very poor for use in real applications. Fortunately, there is a great deal of regularity in language, so that its intrinsic dimension is much lower. Different simplifying assumptions can be viewed as different ways to capture the intrinsic variability in language - that is, identify the meaningful patterns without fitting the noise in a finite training sample. In this thesis we develop a new way to view the sequence modeling problem, which has two key components:

1. We parametrize our models with matrices or tensors to capture bilinear or multilinear relationships between symbols, increasing the expressiveness of our model relative to standard models.

2. We regularize the matrices or tensors to be low rank, sparse, or the sum of the two structures. This yields highly interpretable models. The low rank weights can be seen as finding one or more low-dimensional subspaces that capture the meaningful variation in language, and more directly tackles the goal of uncovering the intrinsic dimension in the high ambient dimension problem. They typically induce continuous low-dimensional representations of words, giving us geometric intuition into the relationships between words. We also observe that sparse weights nicely complement the

low-rank weights, distinguishing exceptional patterns from regular ones.

These characteristics are embodied in three new language models, evaluated under many experimental conditions. In Chapter 3, we introduce a novel factored low rank tensor language model, termed the Low Rank Language Model (LRLM), which is motivated by the observation that most existing language model smoothing techniques have the effect of reducing the rank of conditional probability tables. The LRLM models the joint probability over n-grams directly as a tensor and then constrains the tensor rank. This can be interpreted as learning a mixture of position-dependent unigram models, where the number of mixtures is equal to the tensor rank. We observe that it improves perplexity over the popular, widely-used modified Kneser-Ney n-gram model in a limited-data scenario. We also draw connections between low rank tensor language modeling and related problems, including non-negative tensor factorization.

In Chapter 4, the rank regularization idea is extended into the exponential language model framework, penalizing the rank of a weight matrix rather than the probabilities themselves, which is the fundamental difference relative to LRLM of Chapter 3. The model, termed the Sparse Plus Low Rank Language Model (SLR-LM), gives probabilities $p(x|h)$ of words x given histories h, and the weight matrix defines a bilinear function of word and history features. In doing so, we observe that learning a rank r weight matrix is equivalent to learning r-dimensional continuous representations of words and histories. In contrast to Latent Semantic Analysis, neural network language models and other continuous-space techniques, the rank and thus dimensionality r is learned during training, instead of being fixed ahead of time. One important finding of this work was that rather than using a low rank weight matrix alone, it is better to parameterize the model with the sum of two matrices: the low rank matrix and a sparse matrix. The latter "corrects" the former, increasing or decreasing n-gram probabilities due to exceptional linguistic behavior (e.g. to correct for multiwords like "united states"). We extensively evaluate the SLR-LM, including language modeling experiments in English, Cantonese, Pashto, Turkish, Tagalog and Vietnamese, finding consistent perplexity reductions of 5-9% over a modified-Kneser-Ney baseline. Taking advantage of the flexibility afforded by the feature functions, the chapter includes experimental results using morphological features in Turkish, a morphologically-rich language, where we find that we can improve perplexity over a word-feature-based SLR-LM. Finally, we introduce a new technique for the automatic identification of words and multiwords, using the entries of the sparse matrix, and evaluate it on character-segmented Cantonese data and syllable-segmented Vietnamese data. We find that we can obtain small improvements in perplexity using the learned words, and obtain larger gains in the dictionary precision rates.

In Chapter 5, we extend the SLR-LM to be able to account for overlapping influence on language behavior, including the topic, genre, speaker or author identity, role, etc. This extension, termed the Multi-Factor SLR-LM, replaces the single sparse matrix with a set of sparse matrices, one dedicated to each possible influence. Factoring the model in this way has two key advantages: first, it allows us to identify the words or phrases most important to a given topic, genre, etc., and second, it provides a new method for model adaptation (e.g. certain weights can be kept as is, others can be discarded, and others learned new). We evaluate both on English conversational telephone data, focusing on accounting for variation in topic. We see small gains in perplexity and larger gains in topic keyword precision. We also present a qualitative analysis on a Supreme Court transcript case to explore the potential for capturing more than one type of factor and see that meaningful phrases for speakers, roles and court cases are being learned. Finally, we evaluate the ability to model heterogeneous genres on English broadcast news and broadcast conversation data. Some genre phenomena are captured, but the results motivate an extension of the Multi-Factor model that permits factors to be modeled with low rank corrections, to be left for future work.

Although our focus is on language modeling, in Chapter 6, we introduce new models and training algorithms that extend our approach to two other important human language processing tasks. Our first extension generalizes our sparse plus low rank sequence model to the sequence tagging task, where we need to assign probabilities to one (label) sequence given a second sequence. To do this, we need to extend the bilinear relationships we modeled with matrices to multilinear relationships modeled with tensors. Our second extension branches out into the task of acoustic modeling, where we introduce several ways in which we can use low n-rank tensors to find posterior probabilities over context-dependent phone states given an acoustic feature input.

7.1.2 Specific Contributions

The specific contributions of this thesis are:

- The introduction of the novel factored tensor Low Rank Language Model (LRLM) and the implementation of its learning algorithm, and the interpretation of the LRLM as a mixture of unigrams, drawing a connection to non-negative tensor factorization.

- The introduction of the novel exponential Sparse Plus Low Rank Language Model (SLR-LM) and the implementation of its learning algorithm and lattice rescoring.

- A multi-language evaluation of the SLR-LM, including results on English, Cantonese, Pashto, Turkish, Tagalog and Vietnamese.

- A study of the use of morphological features in the SLR-LM on Turkish data, finding that perplexity improvements can be obtained on a morphologically rich language over a word-only SLR-LM.

- A novel method for finding words and multiwords using the sparse entries of a SLR-LM, and a study of its effectiveness on character-segmented Cantonese and syllable-segmented Vietnamese, in terms of its ability to discover dictionary words and its ability to lower language model perplexity through data resegmentation.

- The introduction of the novel exponential Multi-Factor SLR-LM, which uses supervised training to factor the effects of an arbitrary number of overlapping factors (e.g. topic, genre, speaker) and the implementation of its learning algorithm.

- A novel method for keyword extraction, using the sparse entries of a Multi-Factor SLR-LM, and a study of topic keywords in English.

- A novel method for language model adaptation using the Multi-Factor SLR-LM.

- A modeling and algorithmic guide for extending the low rank sequence modeling techniques to sequence tagging and acoustic modeling applications.

7.2 Future Directions

There are several promising future directions that build on the work in this thesis. We discuss them here in terms of three categories: algorithm and computational improvements, extensions to our language modeling approach, and novel models for new applications.

7.2.1 Algorithmic and Computational

New, sophisticated models often come with a computational cost, and the SLR-LM and its variants are no exception. There are two training bottlenecks, itemized below, with the following potential work-arounds:

1. Storing the weight matrices and computing the singular value decomposition (SVD). In our algorithms, the gradients are typically dense and full rank, so they cannot be represented in a compact form and require $d_\phi d_\psi$ entries. Computing the SVD then requires $d_\psi d_\phi r$ operations, where r is the current estimate of the rank. The easiest way to address both the memory and computational problems is to limit the size of the vocabulary and to limit the n-gram order (thus reducing the dimension of the history features), but alternative encoding strategies could also be employed to reduce the dimensions. Chapter 4 introduced two such options: using a morphological feature representation of words and using a "bag-of-x" representation for histories. Rather than indicator encodings of words and histories, one could also start with moderate-dimensional continuous representations learned elsewhere; for example, from a previously trained lower-order or smaller vocabulary SLR-LM. The primary disadvantage of starting with a continuous representation is that it would come at the cost of interpretability of the sparse matrix, whose entries would no longer represent n-grams. To deal with d_ψ in particular, one strategy would be to limit the vocabulary's size by excluding infrequent words, and design the model to be interpolated with or backoff to a full vocabulary n-gram model, as has been done before with neural network language

models. Another strategy would be to use word classes, as is done in Model M and in newer neural network language models. Regarding the SVD itself, we have done some preliminary exploration of SVD speedups using approximate and randomized algorithms [50, 43, 82], observing a small degradation in performance, but we have not tested this systematically. Another possibility for SVD speed-up is to use the "warmstarted" SVD [130], which does not start the SVD computation from scratch, but is instead seeded with the result from a previous iteration. This approach seems promising, but remains unexplored in our experiments.

2. Computing the gradient. Ultimately, HV conditional probabilities must be computed in order to compute the full gradient, where H is the number of unique histories observed in the training data and V is the vocabulary size. We have implemented and informally explored the use of stochastic mini-batch gradients, where a subset of the histories are sampled and all n-grams with those histories are used in training. When the histories are weighted by the n-gram counts to favor more frequent histories, the method appears to give large memory savings at the expense of some minor degradation in perplexity. Another approach would be to parallelize full gradient computation over multiple machines, as opposed to multiple local cores as is currently done. Parallelization over multiple machines would also help to alleviate the memory costs incurred when H and/or V are large.

Finally, we have relied upon variants of accelerated proximal gradient descent algorithms to solve our optimization problems. Although these algorithms have optimal theoretical convergence rates for the general class to which our problems belong, it is conceivable that the special structure of this problem could be exploited to obtain a better rate. Even if not, future advances in algorithms may offer speed improvements by a constant factor, or even better, may be able to reduce the memory complexity of model training.

7.2.2 Modeling Extensions

One very straightforward extension would be to port the idea of a sparse plus low rank form from Chapter 4 into the factored tensor model of Chapter 3. In this case, the sparse elements would be corrections to the joint probability distribution directly. Although this would likely benefit the modeling power, it would also complicate training, which is already problematic due to the non-convexity of the problem. Given these challenges for training, we feel that improvements targeting the exponential models of Chapters 4 and 5 are more likely to provide a gain.

So far our exponential SLR-LM models assume a bilinear relationship between two objects: the next word x and the history h. One could instead break the single history apart, in order to give additional importance to the most recent words in the history, which provide the most predictive information. In this case one would extract separate representations of the immediate history, and learn interactions between all feature vectors through a weight tensor, $\mathcal{A}$.

Recall that $p(x|h)$ is short-hand for $p(x_t|x_1, \ldots, x_{t-1})$ for some unspecified t. If one believes that the most recent m symbols in the history should receive special treatment, one can define $h_{-i} = x_{t-i}$ for $i = 1, \ldots, m$, and let the remainder of the history be $\tilde{h} = x_1, \ldots, x_{t-m-1}$. Let $p(x|h)$ now denote $p(x|h_{-1}, h_{-2}, \ldots, h_{-m}, \tilde{h})$. Let $\mathcal{A}$ be a $(m+2)$-th order tensor, let Ψ map symbols to $\mathbb{R}^{d_\Psi}$, Ω map symbols to $\mathbb{R}^{d_\Omega}$, and let Φ map sequences of symbols to $\mathbb{R}^{d_\Phi}$. One could then introduce a low n-rank tensor version of the SLR-LM as follows:

$$p_{\mathcal{A}}(x|h) = \frac{\exp(\mathcal{A} \times_1 \Omega(h_{-1}) \times_2 \Omega(h_{-2}) \cdots \times_m \Omega(h_{-m}) \times_{m+1} \Phi(\tilde{h}) \times_{m+2} \Psi(x))}{\sum_{x'} \exp(\mathcal{A} \times_1 \Omega(h_{-1}) \times_2 \Omega(h_{-2}) \cdots \times_m \Omega(h_{-m}) \times_{m+1} \Phi(\tilde{h}) \times_{m+2} \Psi(x'))} \tag{7.1}$$

Recall that $\times_i$ denotes multiplication along the i-th mode. If tensor $\mathcal{A}$ is decomposed into the sum of two tensors, $\mathcal{A} = \mathcal{L} + \mathcal{S}$, this directly generalizes the SLR-LM in Chapter 4. Here, the symbol x is mapped with Ψ instead of ψ, the remaining history $\tilde{h}$ is mapped with Φ instead of ϕ, and the new history elements h_{-i} are mapped through a third function Ω. For simplicity, one could let $\Omega = \Psi$, so that single symbols are mapped the same way, whether in the history or predictive position.

Training this new model is analogous to the SLR-LM:

$$\min_{\mathcal{S},\mathcal{L}} \gamma_0 \|\mathcal{L}\|_* + \gamma_1 \|\mathcal{S}\|_1 - LL(\mathbf{D}; \mathcal{S} + \mathcal{L}) \tag{7.2}$$

where LL is the data log-likelihood as before, and $\|\mathcal{L}\|_*$ denotes the tensor nuclear norm of $\mathcal{L}$. This problem can be solved using the CMLE algorithm of [112].

Recall that the tensor nuclear norm is the sum over each mode i of the matrix nuclear norms of the mode i tensor unfoldings.[1] That is, it approximates the minimum $\sum_i r_i$ such that the fibers of mode i live in an r_i-dimensional subspace. This can also be viewed as learning globally-optimal feature transformations for $\Psi(x)$, $\Omega(h_{-1}), \ldots, \Omega(h_{-m})$ and $\Phi(\tilde{h})$ simultaneously. If the first mode has n-rank r_1, then the first history symbol $\Omega(h_{-1})$ is implicitly mapped down to an r_1-dimensional space. Likewise, if the $(m+1)$-th mode has n-rank r_{m+1}, the vector $\Psi(x)$ is implicitly mapped down to r_{m+1} dimensional space, and so forth. Instead of a simple inner product between the low-dimensional representations, they are combined via tensor multiplication with a full rank core tensor, $\mathcal{C} \in \mathbb{R}^{\times_{i=1}^{m+2} r_i}$. The core tensor encodes the interactions between modes, allowing it to capture more sophisticated sequence information than the SLR-LM alone.

The price to be paid for the richer modeling abilities would be in terms of time and space complexity. Training the model would involve unfoldings along each mode, followed by an SVD on each unfolding. Because unfolding does not efficiently represent the low rank tensor structure, a compact representation of the matrices is not immediately apparent. Storing each matrix requires memory linear in the size of the tensor, $O(d_\Omega^m d_\Psi d_\Phi)$, and computing the SVD for the i-th mode requires $O(d_\Omega^m d_\Psi d_\Phi r_i)$ time. The obvious solution is to limit the size of m, d_Ψ, d_Φ and d_Ω. It is expected that a small m (e.g. 1 or 2) should be sufficient, because 1) most of the key information in language is contained in bigrams and trigrams, and 2) the remainder of the history still appears in the $\tilde{h}$ term. The cost of the large ambient feature dimensions may be addressed with the techniques discussed in Sec. 7.2.1.

One final promising direction of this research is to extend our ideas into new modeling problems beyond sequence modeling. Chapter 6 outlines two such extensions in detail. In the first, we generalize the low n-rank exponential tensor language model idea sketched above to the task of sequence tagging, where multilinear relationships are defined not just between symbols in a single sequence, but between symbols in two sequences. Regularizing the weight tensor n-rank would have the effect of learning continuous, low-dimensional representations of each of the symbols involved in the multilinear relationship. The second extension is to

[1]There exist other definitions of the tensor nuclear norm, but we use this one, defined in Chapter 2, throughout.

the realm of acoustic modeling. Here a multilinear relationship is defined between acoustic features and (a potentially structured representation of) a context-dependent phone state label, and regularizing the n-rank of the weight tensor induces low-dimensional continuous representations of the acoustics and of the phonetic state.

7.2.3 New Applications

One natural application direction for our language models is use in automatic speech recognition. As designed, our models can be used in the second pass (lattice rescoring), but are not practical for first pass decoding. Despite our models' intrinsic low-dimension, they factor in a way that is not compatible with "ARPA" style (back-off) n-gram language models, which are used in many speech recognition systems. This limitation is not unique to our models: it is shared by neural network models (including deep and recurrent models) and by some exponential models. Recently, researchers have introduced a strategy for building ARPA-style models from neural network language models [5]. A similar strategy should be applicable to our models. Using our language models in a first pass rather than second pass would improve its ability to make a meaningful impact on speech recognition performance. Whether via decoding or through lattice rescoring, the use of our language models in automatic speech recognition deserves a careful study.

There are additional interesting applications that make use of the interpretability of the weight structure. For example, in [88], Mikolov et al. find that the low dimensional continuous word representations learned by recurrent neural network language models very effectively encode syntactic and semantic regularities. Relations such as male/female are associated with a specific vector offset; e.g., using the representations of "king" - "man" + "woman" gives a vector close to the representation of "queen." They exploit this property to obtain state of the art performance on the SemEval-2012 *Measuring Relation Similarity* task. Given that the log-bilinear model to which our models are most closely related obtains very strong performance, it would be interesting to evaluate the low dimensional representations learned by the SLR-LM for this same purpose.

An obvious extension of the word and multiword learning application discussed in Section 4.6 would be to learn words and multiwords with more than two syllables. There are two ways to approach this. First, one could learn words with n or fewer syllables directly using an n-gram SLR-LM. Second, and more generally, one could iteratively train bigram models and then resegment the training data. Each new bigram model would be capable of learning words one syllable longer than present in the training data. This second style is analogous to the multiword learning method employed in [102].

The new model extensions presented in Chapter 6 open up many new potential applications. First, it would be interesting to apply the sequence tagging model to the grapheme-to-phoneme task. Here, the goal is to find the most likely phone sequence conditioned on an orthographic character sequence. The low n-rank tensor would induce low dimensional representation of phones and characters, exploiting the phonetic and phonotactic regularities of language. Finally, there are numerous applications of the low n-rank tensor acoustic model, which could be used directly in a hybrid speech recognition system, used as a feature dimensionality reduction technique, or as a mechanism for for tying context-dependent phone states for use in either a HMM-GMM or deep neural network system.

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Appendix A
STOP WORD LIST

The following stop word list was used in experiments in Chapter 5.

</s>	˜	they	this
?	‘	them	these
.	=	myself	that
,	+	yourself	those
;	-	himself	a
:	_	herself	an
"	...	itself	the
’	--	ourselves	who
(	’s	yourselves	whom
)	’d	themselves	whose
<	’t	oneself	what
>	’ve	my	which
/	’m	mine	some
!	i	your	somebody
@	me	yours	someone
#	you	his	something
$	he	her	any
%	him	hers	anybody
ˆ	she	its	anyone
&	her	our	anything
*	it	ours	every
[	we	their	everybody
]	us	theirs	everyone

everything	having	anyway	by
each	will	everywhere	despite
all	would	always	de
both	can	nowhere	down
many	cannot	never	during
much	could	aboard	en
more	shall	about	except
most	should	above	for
too	may	across	from
enough	might	after	in
few	must	against	inside
little	do	ago	into
fewer	does	along	lest
less	did	alongside	like
least	done	amid	minus
no	doing	among	near
nobody	here	amongst	next
nothing	there	around	notwithstanding
none	now	as	of
be	then	astride	off
am	where	at	on
are	when	atop	onto
is	how	before	opposite
was	why	behind	out
were	somewhere	below	outside
been	sometime	beneath	over
being	somehow	beside	par
have	anywhere	besides	past
has	anytime	between	
had	anyhow	beyond	

per	versus	nor	so
plus	via	or	than
post	vs.	plus	that
since	with	so	though
through	within	times	’til
throughout	without	v.	till
’til	worth	versus	unless
till	&	vs.	until
to	and	yet	whereas
toward	both	albeit	whether
towards	but	although	which
under	either	because	while
underneath	et	’cause	yes
unlike	less	don	no
until	minus	don’t	not
unto	’n	if	to
up	’n’	neither	
upon	neither	since	